Exponential relaxation of optical emissions in sprites

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Sprite time scales

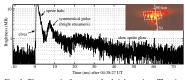


Fig. 1: Photometric features of a bright sprite. The signal from one narrow field-of-view photometer is shown for part of an event at 04:38:27 UT on 19 July 1998. This sprite occurred 0.5 s after a series of intense sprites associated with a series of positive cloud-to-ground lightning discharges. The inset shows a video field of the sprite from the Fly's Eye camera, superposed by a box showing the approximate field-of-view of the photometer and a scale showing altiudes overlying a causative CG.

Exponential relaxation observed

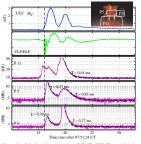


Fig. 2: Lightning/sprite ULF, VLF, and photometric signatures from 6 August 1998. The ULF magnetic field is obtained by integrating in time data (provided by Martin Füllekrug) from a magnetic loop receiver. Photometers 5 and 6 have narrow (2.2°×1.1°) fields-of-view, while the dimensions of Photometer 11 are three times as large. Dashed lines are exponential fits to the curves.

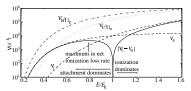


Fig. 3: Model ionization and dissociative attachment rates at 70 km altitude. Also shown are the electron impact excitation coefficients ν_k for the states $B^3\Pi_B$ and $C^3\Pi_u$ producing optical emissions in the N₂(1P) and N₂(2P) bands, respectively. Each rate shown is proportional to atmospheric density, so values at 60 km are ~3.5 times higher than those shown (for 70 km).

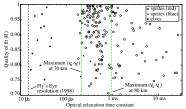


Fig. 4: Exponential decay times in sprites. Values shown as \times 's and o's are from fits giving R > 0.7 using data from one or more red or blue filtered photometers in each of 27 events; each point corresponds to one photometer during one optical pulse of a sprite. The *'s show the result of fits to photometric signatures of elves.

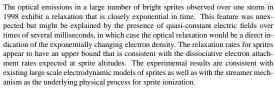
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Abstract

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Macroscopic model

We assume that the system is driven by the lightning current I(t) = dQ(t)/dt which effectively deposits thundercloud charge Q(t) at the altitude h_Q . The total charge which accumulates at the lower end of the sprite is

$$Q_i(t) = Q(t)h_Q/h_i(t)$$

The current flowing through the sprite is therefore

$$I_{\text{sprite}}(t) = \frac{dQ_i}{dt} = I(t)\frac{h_Q}{h_i(t)} - \frac{Q(t)h_Qdh_i}{h_i^2}dt$$

The first term here has to do with the portion of sprite current due to the source (lightning) current, and the second term is due to the descending lower edge of the sprite.

The quasi-constant electric field E can persist in the highly conducting medium of a sprite only of there is a net current flow through the sprite body $I_{\text{Sprite}} \neq 0$. $I_{\text{Sprite}} = \sigma ES$, where σ is conductivity and S is the effective horizontal cross section area of sprite.

Microscopic model

We solve the following set of dynamic equations for number densities of electrons n_e , positive n_+ and negative n_- ions:

$$\frac{dn_e}{dt} = \left(\nu_i(E) - \nu_a(E)\right)n_e$$

$$\frac{dn_+}{dt} = \nu_i(E)n_e \tag{3}$$

$$n_- = n_+ - n_e \tag{4}$$

At the initial moment of time t=0 it is assumed that $n_e=n_+=n_{e,0}$, and $n_==0$, where $n_{e,0}$ is the electron number density of stable streamers derived from similarity laws $(n_{e,0}=5.5\times10^6 \text{ cm}^{-3} \text{ at } 60 \text{ km} \text{ and } n_{e,0}=4.6\times10^5 \text{ cm}^{-3} \text{ at } 70 \text{ km})$. The electric field E in the above equations is assumed to be a function of time determined by current continuity as

$$E = \frac{J(t)}{\sigma} = \frac{J(t)}{eneute + e(n_+ + n_-)u_i}$$

where μ_e and μ_i are mobilities of electrons and ions which in the present model are assumed to be constants. The time dependent streamer current density J(t) is defined at time t = 0 as $J(0) = J_0 = (en_e,\mu_e+en_e,\mu_i)E_k (E_k=761 \text{ V/m} \text{ and } 219 \text{ V/m} \text{ at altitudes } 60 \text{ and } 70 \text{ km}$, respectively), and then assumed to decay as $J(t) = J_0 \tau_I / (t + \tau_I)$. The characteristic time scale τ_I of current decay in the sprite is considered as the only input parameter in our model, with all the rest calculated self-consistently.

Before considering the results of numerical calculations let us qualitatively inspect a case when $J(t)=J_0$ =const. The above equation (5) for the electric field can be rearranged as

$$(en_e\mu_e + e(n_+ + n_-)\mu_i)E = J_o$$

Even a small increase in ion concentration would lead to an increase in the ion conductivity contribution to J_0 and therefore to a drop in the E field. However, even a small reduction in the electric field leads to exponential relaxation of the electron density, which quickly compensates for the increase in the ion conductivity. A remarkable feature of this system is that the electron density relaxes exponentially in time, while the electric field is reduced by a very small amount and essentially remains constant in time with high accuracy.

Summarv

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Quasi-steady currents in sprites are described in terms of two large scale physical processes – the slow variation of either the lightning source current or the descent of the sprite's lower edge. In turn, sprite currents which vary slowly compared with the time scale of electron dissociative attachment to O_2 lead to quasi-constant total conductivity, even while the electron contribution to conductivity is decreasing. This results in a property of streamers driven by strong external electric fields, namely, that the local electric field is maintained self-consistently near the breakdown electric field, E_k .

A quasi-constant electric field just below E_k leads straightforwardly to an exponential relaxation of the electron density and the optical emission rate in sprites. According to our model total ion densities become significant and may dominate the conductivity. Exponential decay of photometric features associated with sprites has been observed in a number of bright sprites and appears to be a form of non-spectroscopic evidence for large ionization changes in sprites. The observed photometric behavior is consistent with the streamer mechanism as the underlying physical process for sprite ionization. In future sprite measurements, more attention could be paid to this phenomenon, in particular in the context of multicolor photometry and photometry with fine altitude discrimination.

 $h_{i}(t) \xrightarrow{q_{i}(t)} 0 \xrightarrow{q_{$

 $\begin{array}{c} h_{\mathcal{Q}^{-}--} \underbrace{-\mathcal{Q}(t)}_{+\mathcal{Q}(t)(l-h_{\mathcal{Q}}/h_{i}(t))} \\ 0 \end{array}$

Fig. 5: Macroscopic model: Diagram of charge systems in sprites.

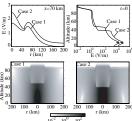




Fig. 6: Macroscopic model: Spatial variation of the model electric field shown at the moment of time 5 ms

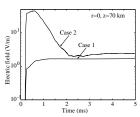


Fig. 7: Macroscopic model: Temporal variation of the model electric field.

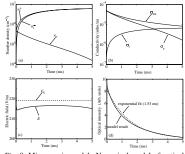


Fig. 8: Microscopic model: Numerical model of optical relaxation. (a) Electron and ion densities for $\tau_{T^{-1}}$ ms; (b) the corresponding conductivities; (c) the corresponding electric field; and (d) resulting optical emissions.

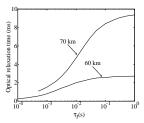


Fig. 9: Microscopic model: Dependence of τ_0 , the optical relaxation time constant, on τ_I , the time scale of change in the sprite current.

References

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