Why Should Latin Squares Have All the Fun?

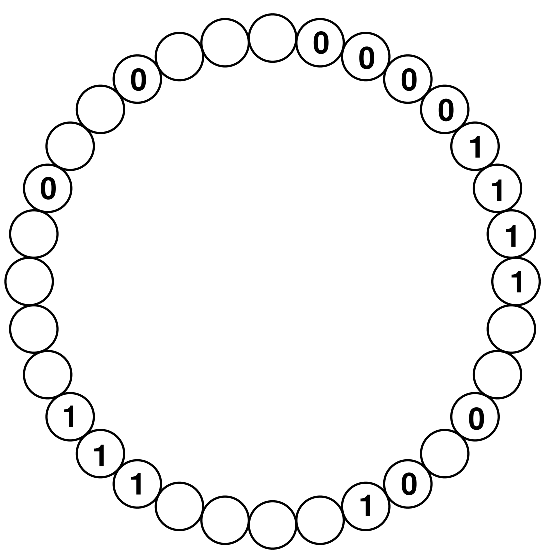
If you're looking for a change of pace from Sudoku, try the following variants, which have a similar flavor but are based on combinatorial objects other than Latin squares.

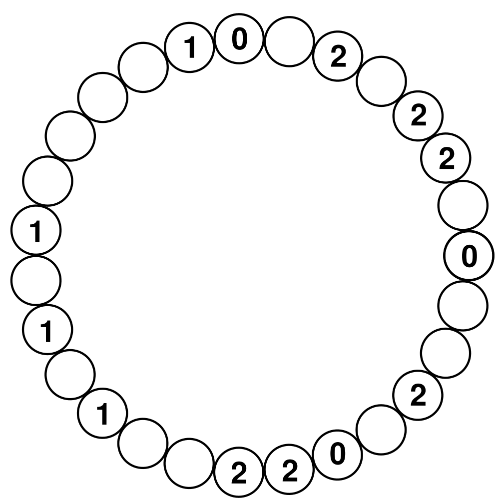
In the first two puzzles, you must complete the *Latin tableau.* Each row must contain some permutation of the numbers from 1 to r, where r is the length of that particular row, and each column must contain some permutation of the numbers from 1 to c, where c is the height of that particular column.

			6				5		
	7								2
		9				1			
8				3	6				
4									
				5					
	3								
		1							

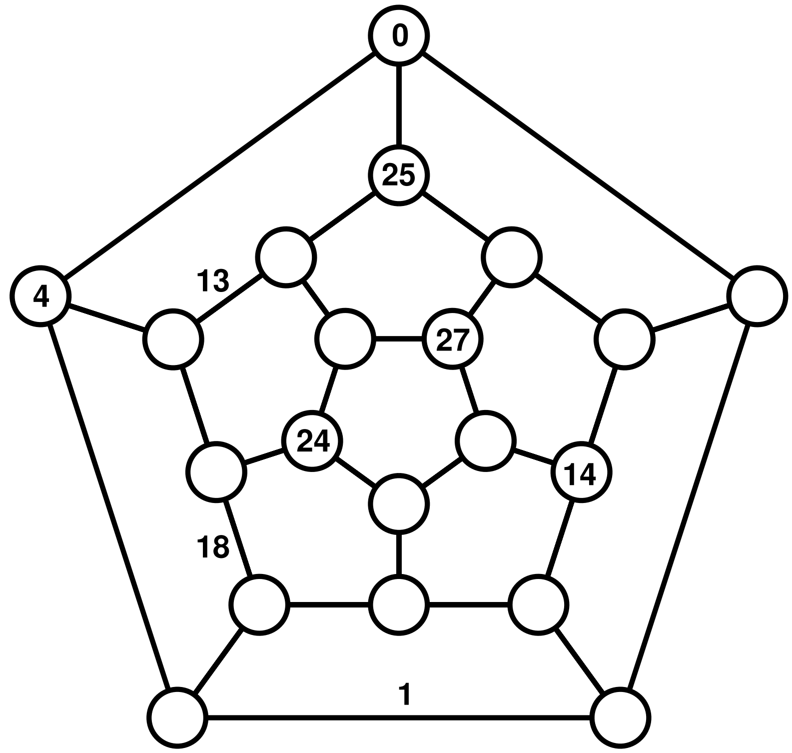
						5			2
								1	
10					6				
		5	8			2			
6		1							
	6				1				
					4				
		3							
1									
			3						

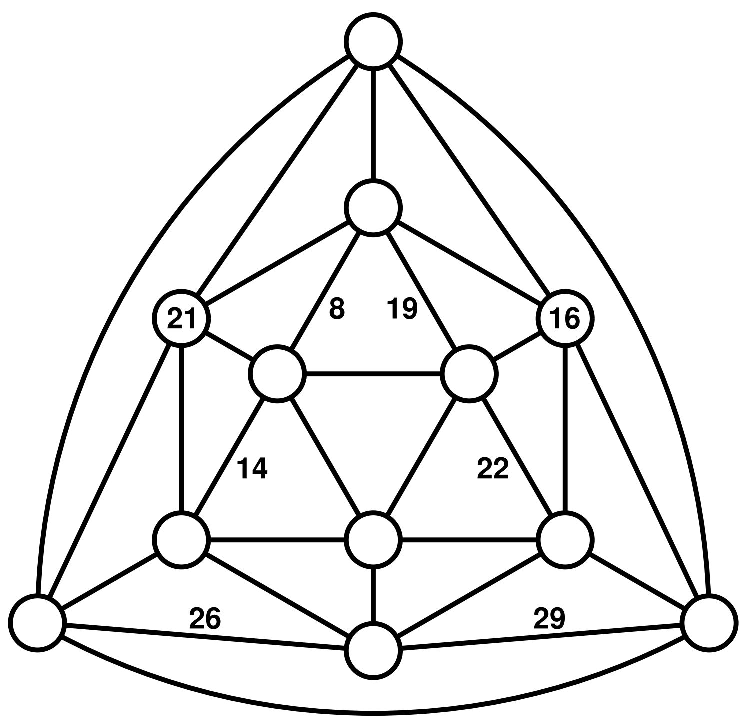
In the next two puzzles, you must complete the *de Bruijn sequence.* Each of the b^n possible n-digit numbers in base b (leading zeroes are allowed) must occur exactly once as a [clockwise] subsequence of n consecutive digits somewhere on the cycle. Different n-digit numbers may overlap. In the left-hand puzzle, $b = 2$ and $n = 5$; in the right-hand puzzle, $b = 3$ and $n = 3$.





In the next two puzzles, you must complete the *graceful labeling*. Each vertex is labeled with a distinct nonnegative integer from the range 0 to e, where e is the number of edges of the graph (not every possible vertex label will be used). Each edge is labeled with the difference between the labels of its endvertices, and each edge label $1, 2, \ldots, e$ must arise exactly once.





In the seventh and last puzzle, you must complete the *Kirkman triple system.* In each of the seven 3×5 grids, each of the numbers 1 to 15 must appear exactly once, and for each (unordered) pair of numbers from 1 to 15—there are 105 such pairs—there must be some grid that puts both of those numbers in the same row. There is no constraint on the columns; in particular, in any row of any grid, the ordering of the three numbers in that row is immaterial. This freedom has been used to spell Gardner's name.

	15	9
5		
10		14
12		7
	2	3

4	11	
		8
	6	12
2		5
	13	14

	7	
9		3
14	12	
5	1	
11		13

		6
		7
2	15	13
4		8
	10	11

	10	
6		1
15		3
13		9
14		5

	6	8
10		15
4	3	
9		
	1	13

	1	
7		10
11	8	
12	9	
2		4

Notes and References

It is an open problem to characterize the [Ferrers] shapes for which a Latin tableau is possible. A necessary condition is given, and conjectured to be sufficient, in T. Y. Chow, C. K. Fan, M. X. Goemans, and J. Vondrak, "Wide partitions, Latin tableaux, and Rota's basis conjecture," *Advances Appl. Math.* **31** (2003), 334–358.

De Bruijn sequences are so called because of N. G. de Bruijn's pioneering paper "A combinatorial problem," *Indagationes Math.* **8** (1946), 461–467, and are now a standard topic in discrete mathematics and computer science.

Martin Gardner wrote about graceful labelings in *Scientific American* **226** 3 (1972), 108–112; **226** 4 (1972), 104; **226** 6 (1972), 118. Readers found graceful labelings of the icosahedron and the dodecahedron, but explicit labelings have apparently never been published. J. Gallian maintains a dynamic survey on graceful labelings and other graph labeling problems in the *Electronic J. Combin.* (`http://www.combinatorics.org/Surveys/ds6.pdf`).

The Kirkman triple system is a fitting topic for G4G7 because not only are there seven 3×5 blocks, but there are seven essentially distinct solutions to the famous Kirkman schoolgirl problem (T. P. Kirkman, "On a problem in combinatorics," *Cambridge and Dublin Math. J.* **2** (1847), 191–204; R. J. R. Abel and S. C. Furino, "Kirkman triple systems," in *The CRC Handbook of Combinatorial Designs*, ed. C. J. Colbourn and J. H. Dinitz (CRC Press, 1996), §I.6.3, pp. 88–89).

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Dedicated to Martin Gardner, G4G7, March 2006