Algorithm for traffic grooming in optical networks to minimize the number of transceivers

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Abstract— **In this paper, we study the problem of traffic grooming to reduce the number of transceivers in optical networks. We show that this problem is equivalent to a certain traffic maximization problem. We give an intuitive interpretation of this equivalence and use this interpretation to derive a greedy algorithm for transceiver minimization. We discuss implementation issues and present computational results comparing the heuristic solutions with the optimal solutions for several small example networks. For larger networks, the heuristic solutions are compared with known bounds on the optimal solution obtained using integer programming tools.**

I. Introduction

It is widely believed that "All Optical Networks" with *wavelength division multiplexing (WDM)* will be the future wide-area backbone networks. A WDM network consists of several nodes interconnected with fiber optical (physical) links. The traffic signals in the network propagate through the optical fiber at different wavelengths and therefore the network can be alternatively thought of as a set of nodes interconnected by *lightpaths.* A lightpath is a path of physical links in which a particular wavelength on each link is reserved for the lightpath. The traffic signals in the network remain optical (or almost optical) throughout their flow through a lightpath. We consider networks with fixed (static) full-duplex lightpaths. The lightpaths are terminated at each end by transceivers which are optoelectronic equipment that convert the optical signals into electronic signals for further processing. The cost of the transceivers is a dominant cost in the network. Therefore, in this paper we study network design to minimize the number of transceivers. Since the number of transceivers is twice the number of lightpaths, minimizing the number of transceivers is equivalent to minimizing the number of lightpaths.

We assume that the networks are circuit-switched and support lower-speed full duplex end-to-end connections or traffic streams, all at same rate. We also assume that all lightpaths have the same transmission capacity measured in the units of traffic streams. For example, if each traffic stream is an OC-3 connection and a lightpath is an OC-48 connection, we will say that capacity of the lightpath is 16 traffic streams. There are two topologies associated with such WDM optical networks. They are

1. Physical topology, determined by the set of physical links, and
2. Virtual topology, determined by the set of lightpaths or logical links.

Note that several virtual topologies can be implemented on a physical topology and that not all virtual topologies are supported by a given physical topology. Moreover, the number of transceivers in a network depends only on the virtual topology, whereas other system parameters like performance, number of wavelengths, etc., depend on how the lightpaths are implemented on the given physical topology. Since we are interested only in the cost of transceivers, it is sufficient to restrict ourselves to virtual topologies in this paper. In particular, our algorithms prescribe only a virtual topology. They do not describe how the virtual topology is realized on the physical network.

The general problem of designing a WDM network (i.e., designing virtual topology on a given physical topology) to optimize system cost and system performance is known to be an NP-hard problem. In this paper, we study a very special case in which all costs except the cost of transceivers are neglected. It is also assumed that all virtual topologies are implementable on the given physical topology. Although this assumption is very restrictive, it is satisfied in the following situation.

In wide-area optical WDM networks, if the number of physical links (fibers) available between neighboring nodes is large, then the number of wavelengths in the system is not important and two lightpaths can always be routed on two different physical links even though they use the same wavelength. Moreover, if any two nodes can be connected by a path of physical links, then between any two nodes any number of lightpaths can be implemented.

With these assumptions, the cost in the network will be concentrated mainly in the transceivers. The number of transceivers used in the system can be reduced by careful aggregation of traffic streams on to lightpaths. In the next section,

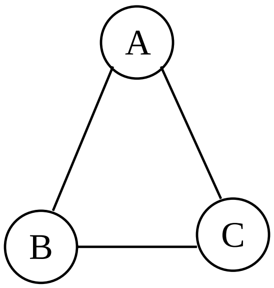

Fig. 1. Example network

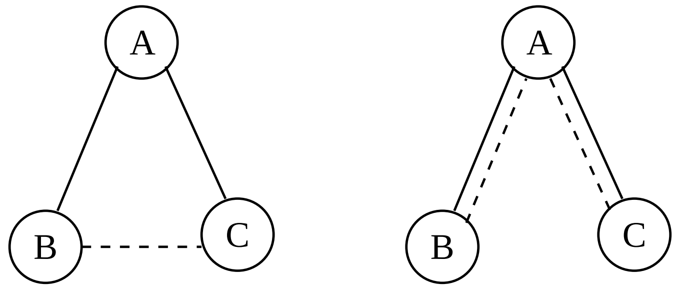

Fig. 2. Two solutions

we motivate this transceiver minimization problem with an example and give a precise formulation of the problem. In the subsequent sections, we discuss our heuristic and present computational results.

The transceiver minimization problem can be thought of as a very special case of virtual topology design of optical networks [1]. Typically, researchers have concentrated on network design to optimize number of wavelengths, delay, probability of blocking and congestion. References [2], [3], [4] were the first to consider transceiver costs. They decompose the design of WDM ring networks into two phases. In the first phase, the low-speed traffic streams are aggregated on to lightpaths to minimize the number of transceivers and the number of wavelengths. In the second phase, these lightpaths are assigned wavelengths. In contrast, we consider only the first design phase and restrict ourselves to minimizing transceiver costs which depend only on how the traffic streams are aggregated on to lightpaths.

Although we present our algorithms in the context of optical networks, they can be applied to any networks involving two layers (e.g., electronic and optical in optical networks) and when transceivers are required to aggregate several traffic streams of one layer into a traffic stream of the other layer.

II. Motivation and Problem Statement

As an example, consider a network with three nodes A, B and C with the physical topology as shown in Fig. 1. With this physical topology, assuming that the number of fibers is not a constraint, lightpaths between any two nodes in the network can be implemented. Assume that we need to build a network with one traffic stream between each pair of nodes and the capacity of each lightpath is 2 traffic streams. There are two different ways of aggregating the traffic streams as shown in Fig. 2. One way is to have a lightpath between every pair of nodes and route the traffic such that it takes the shortest path of lightpaths. This solution uses six transceivers. In the second solution, the network has only two lightpaths AB and AC. The traffic between nodes A and C is routed through B. Between B and A the two traffic streams AB and BC share the same lightpath and similarly, between A and C the traffic streams AC and BC share the same lightpath. This solution saves one lightpath and two transceivers. This example shows that careful aggregation of low speed traffic streams on to lightpaths can decrease the cost of the transceivers used in the network. This decrease in cost, compared to the cost of obvious solutions, can be substantial in large networks. In the following subsection, we formally state this problem of minimizing transceiver cost.

A. Problem Statement

Given a traffic pattern and the capacity c of the lightpaths in the network, the problem is to design a virtual topology with least possible number of lightpaths and route each traffic stream through these lightpaths. In an n-node network, the traffic pattern can be represented by an $n \times n$ matrix $T = [T_{ij}]$, where T_{ij} equals the number of traffic streams from node i to node j. Similarly, the virtual topology of the network can be represented by an $n \times n$ matrix $L = [L_{ij}]$, where L_{ij} equals the number of lightpaths from node i to node j. Note that T and L are symmetric matrices as lightpaths and traffic streams are full-duplex. The routing of the traffic streams T through a virtual topology with lightpaths L can be represented by non-negative integers f_{ijkl} , for each i, j, k, l where f_{ijkl} equals the number of traffic streams from node k to node l routed through the lightpath from node i to node j. It is easy to see that f_{ijkl} should satisfy the following constraints.

1. The constraint that each lightpath can carry at most c traffic streams can be expressed as: for each i, j,

$$\sum_{k,l} f_{ijkl} \le cL_{ij}. \tag{1}$$

2. Similarly, the constraint that traffic streams and lightpaths are full-duplex can be expressed as: for each i, j, k, l,

$$f_{ijkl} = f_{jilk}. \tag{2}$$

3. Finally, the constraint that traffic flowing into a node k is equal to the traffic flowing out of the node k plus the traffic dropped at that node can be written as: for each k, l, m,

$$\sum_i f_{iklm} - \sum_i f_{kilm} = b^k_{lm}, \tag{3}$$

where

$$\begin{aligned} b^k_{lm} &= 0 \quad \text{if} \quad k \neq l \quad \text{and} \quad k \neq m, \\ &= T_{lm} \quad \text{if} \quad k = m, \\ &= -T_{ml} \quad \text{if} \quad k = l. \end{aligned}$$

The problem is to minimize $\sum_{i,j} L_{ij}$ subject to constraints (1),(2) and (3) for a given matrix T. This is an integer linear program (ILP) with $O(n^4)$ variables. This ILP is a special case of multicommodity flow problems [5] which become unmanageable even for moderate sized networks (say 20 nodes). Therefore, we have to resort to heuristics to obtain "good" solutions in a reasonable amount of time. In the next section, we develop one such heuristic based on an intuitive observation which we call "duality".

Given a traffic pattern T one obvious virtual topology through which the traffic can be routed is the following. Between each pair of nodes i and j, $\left\lceil \frac{T_{ij}}{c} \right\rceil$ lightpaths are used and all the traffic streams between these nodes are routed through these lightpaths. This solution is viable if the number of traffic streams T_{ij} is less than but close to capacity c. On the other hand, if the difference $c - T_{ij}$ between the capacity of the lightpath and the part of it used by the traffic streams is large this solution becomes very expensive as a lot of capacity is wasted. To decrease the cost, some of the lightpaths can be removed from this virtual topology and the traffic streams routed through these lightpaths can be rerouted through the surplus capacity in the remaining lightpaths. In our heuristic, we start with an initial network and delete lightpaths one by one and reroute the traffic in the deleted lightpaths until no more deletions are possible.

III. Duality and Heuristics

In order to understand duality, we need a further abstraction of the networks. A network can be viewed as a set of nodes interconnected by capacities represented by a matrix $C = [C_{ij}]$, where C_{ij} is the capacity between nodes i and j. For example, the capacities of a network with virtual topology L are represented by the matrix cL. Note that, whether or not a given traffic pattern can be routed through a network depends only on the capacities in the network. Suppose traffic patterns T and T' can be routed through capacities C and C' respectively. Then, it is easy to see that the traffic pattern $T + T'$ can be routed on capacities $C + C'$. Using this observation, we state our principle of duality in the following subsection.

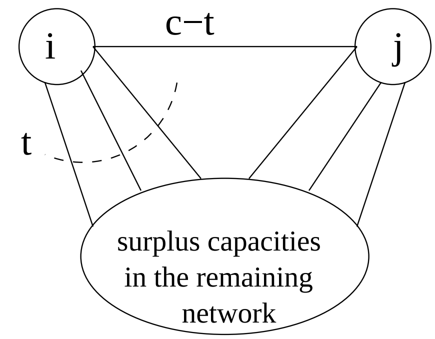

Fig. 3. Loss in surplus capacities

A. Duality

Suppose the traffic pattern T can be routed through capacities C. Let $K = [K_{ij}]$ be a matrix such that $K_{ij} \geq C_{ij} + T_{ij}$ for all i and j. Then the traffic pattern $K - C = (K - C - T) + T$ can be routed through the capacities $K - T = (K - C - T) + C$.

Using capacities instead of virtual topology, we can rewrite the transceiver minimization problem as:

Problem 1: Minimize $\sum_{i,j} L_{ij}$ subject to the constraint that the traffic pattern T can be routed on capacities cL.
Using variable transformations $D_{ij} = \tilde{L}_{ij} - L_{ij}$, for sufficiently large matrix $\tilde{L}$, and the above duality, this problem can be rewritten as:

Problem 2: Maximize $\sum_{i,j} D_{ij}$ subject to constraint that the traffic pattern cD can be routed on capacities $c\tilde{L} - T$.
This dual problem has the following interpretation. Consider a network with lightpaths $\tilde{L}$. Since $\tilde{L}$ is large, we can route each traffic stream through the lightpaths between its terminal nodes. This routing leaves capacities $c\tilde{L} - T$ unused, and we call these the surplus capacities. Suppose we wish to remove a lightpath carrying t traffic streams and reroute this traffic. This rerouting is possible only if we can route t traffic streams through the surplus capacities in the *remaining* network. The decrease in surplus capacities due to removal of the lightpath and rerouting of the traffic streams is as shown in Fig. 3. Note that these capacities correspond to those used by some routing of c traffic streams through the surplus capacities between nodes i and j. From this discussion, it is easy to deduce that the removal of a lightpath between nodes i and j and rerouting of traffic streams carried by this lightpath is possible if and only if c traffic streams can be routed through surplus capacities $c\tilde{L} - T$ between nodes i and j. More generally, we can delete lightpaths represented by matrix D from the network with lightpaths $\tilde{L}$ and reroute the traffic carried by these lightpaths if and only if cD traffic streams can be routed through surplus capacities $c\tilde{L} - T$. Therefore, the dual problem can be viewed as starting

with an initial network with $\tilde{L}$ lightpaths and removing as many lightpaths as possible in the network and rerouting the traffic in these deleted lightpaths. This view immediately suggests the following greedy heuristic to minimize lightpaths.

B. Heuristic

1. Start with a network with $L := \tilde{L}$ lightpaths for some large $\tilde{L}$. Use the shortest path routing of traffic streams through these lightpaths. With this routing the surplus capacities in the network is equal to $c\tilde{L} - T$.
2. While the surplus capacities support c traffic streams between some pair of nodes repeat steps 3, 4 and 5.
3. Find the pair (i, j) of nodes between which the shortest path routing of c traffic streams through surplus capacities uses least amount of the total capacity.
4. Delete a lightpath between nodes i and j in the network and reroute all the traffic through this lightpath through the surplus capacities used in the Step 3, i.e., let $L_{ij} := L_{ij} - 1$.
5. Delete the capacities used by Step 3 from the current surplus capacities.

Note that Step 3 involves enumerating all pairs of nodes (i, j) and solving a minimum cost flow problem for each pair. Although, the way we presented our algorithm suggests that we have to keep track of the route of each traffic stream throughout the algorithm, this is not necessary as the algorithm moves from one shortest path routing to another. One can delete the lightpaths from the network with out worrying about how these deletions affect the routes of individual traffic streams. When the final network is obtained, the route of each traffic stream can be obtained by shortest path routing which is a solution to a minimum cost network flow problem. The number of iterations performed by the above heuristic depends on how large the initial set of lightpaths $\tilde{L}$ is chosen. Since, the greedy heuristic deletes all lightpaths that do not carry any traffic streams before deleting a lightpath that carries some traffic stream, it is optimal to choose the initial network with $\tilde{L}_{ij} = \left\lceil \frac{T_{ij}}{c} \right\rceil$, where $\lceil \cdot \rceil$ represents the ceil function. Finally, we would like to point out that the above heuristic can be applied using different selection strategies for Step 3, and can be extended to more complicated problems, e.g., with transceiver costs dependent on nodes.

IV. Computational Experiments

In this section, we compare solutions obtained by applying our heuristic to some example traffic patterns with the optimal solutions. The optimal solutions were obtained using integer linear programming tools of CPLEX 6.0. For some patterns, these tools were unable to solve the problem. For such problems, our solutions are compared with the bounds obtained using these tools.

A. Uniform traffic pattern

The following table shows computational results for an 8-node network with 8 units of capacity for each lightpath. The traffic pattern is uniform in the sense that there are same number of traffic streams between any two nodes.

Traffic	Optimal	Heuristic
1	9	10
3	17	18
5	23	24

The following table shows results for a 10-node network with capacity of 8 units for each lightpath.

Traffic	Optimal	Heuristic
3	25	28
5	35	37

B. Random traffic pattern

The following table shows results for networks for which the traffic patterns have been generated randomly.

Number of Nodes	Fiber Capacity	Optimal	Heuristic
8	8	18	20
16	24	<122, >96	106

V. Conclusions

The computational experiments clearly show that our algorithm does not give optimal solutions. However, we do not understand well why this is so. Moreover, we do not know whether the transceiver minimization problem is NP-hard. There are no known bounds on the performance of the above heuristic compared to the optimal solution.

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