LINEAR-PROGRAMMING DESIGN OF DATA-COMMUNICATION PULSES TOLERANT OF TIMING JITTER OR MULTIPATH

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ABSTRACT The amount of multipath spread or timing jitter that a data-communication signal can tolerate depends on the shape of the receive-filter output eye pattern around the optimal sampling instant. Ideally, the height of the eye opening would remain unchanged as the sampling instant deviated from the optimum. If there is enough excess bandwidth to permit the necessary design control over the shape of the system pulse, a practical system can be designed that is quite close to this ideal. Such high-bandwidth systems might be found in multipath-prone environments or in "fall-back" modes of high-speed systems, for example. Here example designs demonstrate linear-programming optimization strategies that use that bandwidth to make the eye opening flatter near the optimal sampling instant for an hybrid FIR/analog pulse-shaping system subject to spectral-mask constraints.

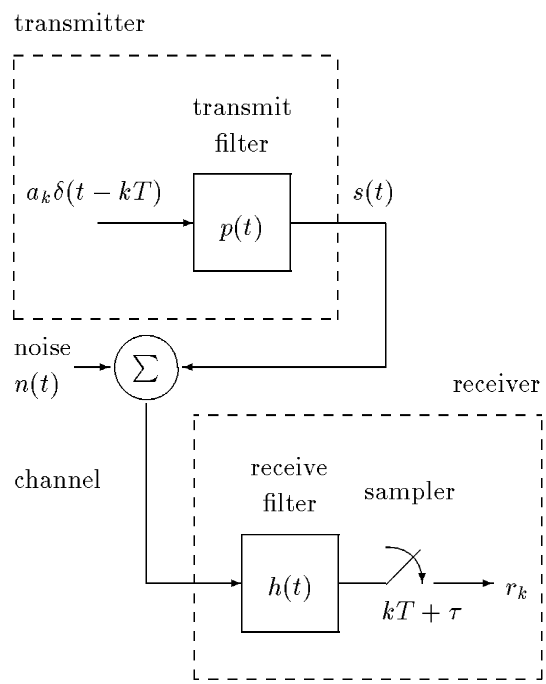


Figure 1: Linear baseband signaling system.

I INTRODUCTION

A *canonical linear program* comprises a finite collection of *canonical optimization variables* $x_1, \ldots, x_n$, a finite collection of inequality constraints of the form $\alpha_1 x_1 + \cdots + \alpha_n x_n \le c$, and an *objective* $\beta_1 x_1 + \cdots + \beta_n x_n$ where $\alpha_1, \ldots, \alpha_n$, c, and $\beta_1, \ldots, \beta_n$ are real constants. A *solution* of the linear program is a set of nonnegative, real values for $x_1, \ldots, x_n$ that results in the largest value possible for the objective given that the constraints must all be met.

This paper explores the linear-programming design of data-communication pulses when the goal is to optimally put excess available bandwidth to use to increase tolerance of timing jitter and multipath. The increased tolerance to these system disturbances results from a flatter eye pattern around the optimal sampling instant. Because bandwidth is the key, this approach is of most use in special-purpose wideband systems, but the same techniques can benefit the design of narrowband systems as well by putting excess degrees of optimization freedom to work to design the eye opening to be a little flatter at a negligible cost in other performance measures. The automation of the more tedious parts of linear-program formulation with FF [1, 2], a special-purpose linear-programming design language, makes it relatively easy to explore design strategies.

To lay a foundation for the subsequent development of eye-flattening techniques, Part II begins with a discussion of the basic linear-programming pulse-design philosophy developed previously [1–3] and concludes with a detailed FF pulse-design example that will later serve as a basis for performance comparison when the design is modified to flatten the eye. The design of jitter- and multipath-tolerant pulses is examined in Part III, where the development of the relevant theory is followed by detailed design examples that demonstrate two strategies for making the eye opening flatter near the optimal sampling instant.

II BASIC PULSE DESIGN

A Intersymbol Interference Basics

In the Time Domain Figure 1 shows a simple linear baseband signaling system. The response $p \circledast h$ of the receive filter to the transmitted pulse is referred to as the *system pulse.* Zero intersymbol interference requires that the this system pulse go through zero at offsets from τ that are nonzero multiples of the signaling interval. This has a salutary effect on the sampled receiver output:

$$r_k = \left[\left(\sum_m a_m p(t-mT) + n(t) \right) \circledast h(t) \right]_{t=\tau+kT}$$

or

$$\begin{aligned} r_k &= \sum_m a_m (p \circledast h)(\tau + (k-m)T) \\ &\quad + (n \circledast h)(\tau + kT) \qquad (1) \\ &= a_k (p \circledast h)(\tau) + (n \circledast h)(\tau + kT). \end{aligned}$$

Each transmitted pulse contributes to a single receiver output sample. For convenience of notation, define system gain $\kappa \triangleq (p \circledast h)(\tau)$ and a normalized and sampled system pulse $q_k \triangleq \kappa^{-1}(p \circledast h)(\tau + kT)$, in order to obtain this more-useful equivalent of (1):

$$r_k = \kappa \sum_m a_m q_{k-m} + (n \circledast h)(\tau + kT), \qquad (2)$$

So, the transmit and receive filters together display zero intersymbol interference whenever $q_k = \delta_k$, a Kronecker delta, or equivalently when $(p \circledast h)(\tau + kT) = 0, \forall k \neq 0$.

The Nyquist Criterion This zero-ISI requirement on the system pulse can be expressed in the frequency domain: the discrete-time Fourier transform of the requirement $q_k = \delta_k$ gives $Q(f) = 1$, which is equivalent to[1]

$$\frac{1}{T} \sum_k P(u) H(u) e^{2\pi i u \tau} \big|_{u=f-\frac{k}{T}} = \kappa \qquad (3)$$

which is sometimes called *Nyquist's second criterion,* for historical reasons. If the receiver is matched, so that $h(t) = p^*(\tau - t)$, then $H(u) = P^*(u) e^{-2\pi i u \tau}$, and Nyquist's criterion becomes

$$\frac{1}{T} \sum_k |P(f - \frac{k}{T})|^2 = \kappa. \qquad (4)$$

When this condition holds for $P(f)$, the transmit pulse is said to be *root-Nyquist,* because its squared magnitude satisfies a Nyquist condition. When designing a root-Nyquist transmit pulse, it is convenient and harmless to assume that $\tau = 0$, and I shall do so below. Realistic values of τ are only relevant in the design of the receive filter.

Normalized Peak ISI There are many ways to characterize the degree of ISI suffered by a system. Two of the simplest and most intuitively appealing are rms ISI and peak ISI. Rms ISI is useful because when peak ISI is much smaller than rms noise (AWGN) at the output of the receive filter, the effect of ISI on symbol error rate can be estimated by adding mean-square ISI to mean-square noise to treat ISI as noise. When peak ISI is much larger than rms noise, however, the *peak* ISI determines the effect of ISI on error rate. Peak ISI is also amenable to constraint by a linear program, whereas rms ISI is not, and so only peak ISI is considered here

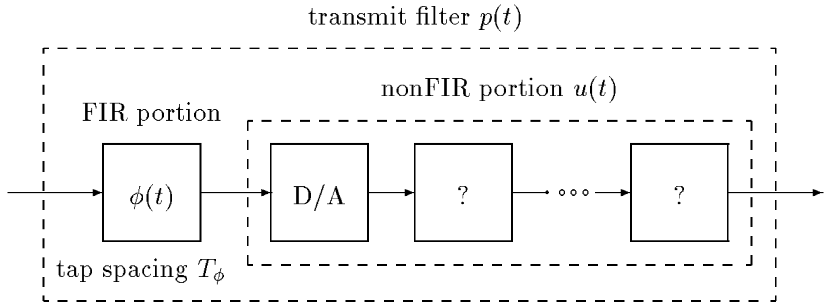
Since $q_0 = 1$, q_0 can be split from q_m by adding and subtracting a Kronecker delta: $q_m = \delta_m + (q_m - \delta_m)$. When the convolution in a noise-free version of (2) is written $r_k = \kappa \sum_m q_m a_{k-m}$, this allows the receiver output to be split into the desired component and the ISI:

$$r_k = \underbrace{\kappa a_k}_{\text{desired}} + \underbrace{\kappa \sum_m (q_m - \delta_m) a_{k-m}}_{\text{ISI}}.$$

Peak ISI is simple to compute only in the special case in which q_k is real,[2] and in which

[1] If the left side of this equation is equal to a constant, then it will in fact be equal to κ, because of the way κ is defined.

[2] In a trivial generalization, αq_k is real for some complex constant α.



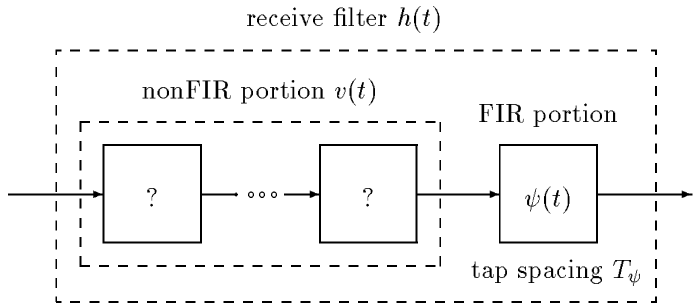


Figure 2: The transmit and receive filters of Fig. 1 each comprise FIR and nonFIR portions.

each complex symbol a_k is drawn from a constellation A that is *balanced* in the sense that $a \in A \Rightarrow -a \in A$. Then if the symbols $\{a_k\}$ can be chosen to make the identity $|\sum_m (q_m - \delta_m) a_{k-m}| \leq \sum_m |(q_m - \delta_m)||a_{k-m}|$ hold with equality, the magnitude of the ISI will be as large as possible. Suppose $a \in A$ has the largest magnitude of any element of constellation A. Choose $a_{k-m} = a\,\mathrm{sgn}(q_m)$, so that $\kappa \sum_m (q_m - \delta_m) a_{k-m} = \kappa \sum_m (q_m - \delta_m)\,\mathrm{sgn}(q_m) a = \kappa a \sum_m |q_m - \delta_m|$. Since κa will have the largest magnitude possible for the desired signal,

$$\frac{\text{largest ISI}}{\text{largest desired signal}} = \sum_m |q_m - \delta_m|, \quad (5)$$

which is then a suitably normalized measure of peak ISI. This quantity is directly constrained in the example designs below.

Average Transmitted Power If the the symbol sequence can reasonably be assumed to be uncorrelated with zero mean, the power spectral density of $s(t)$ is given by

$$S_s(f) = (\sigma_a^2/T)|P(f)|^2. \quad (6)$$

where σ_a^2 is the (stationary) variance of symbol a_k. The example designs will use this spectrum.

If in addition $P(f)$ can be considered for practical purposes to be bandlimited to $|f| < T^{-1}$, then a remarkably simple and useful expression for the total transmitted power P_{tx} results if the transmit pulse is root Nyquist:

$$\begin{aligned} P_{\mathrm{tx}} &= \frac{\sigma_a^2}{T} \int_{-1/T}^{1/T} |P(f)|^2\, df \\ &= \frac{\sigma_a^2}{T} \int_{[0,T^{-1})} |P(f)|^2 + |P(f-1/T)|^2\, df. \end{aligned}$$

Since there is zero ISI, (4) holds. Further, the bandlimiting ensures that the sum in (4) has only a single nonzero term when $f = 0$, so that

$$\kappa = \frac{|P(0)|^2}{T}. \quad (7)$$

For $0 \leq f < T^{-1}$, there are at most two nonzero terms, so

$$\begin{aligned} |P(f)|^2 + |P(f-1/T)|^2 &= \kappa T \\ &= |P(0)|^2. \quad (8) \end{aligned}$$

The transmitted power now becomes

$$P_{\mathrm{tx}} = \frac{\sigma_a^2 \kappa}{T} = \frac{\sigma_a^2 |P(0)|^2}{T^2}. \quad (9)$$

These expressions will prove useful in the example designs to follow.

The Half Symbol-Rate Point For a bandlimited root-Nyquist pulse, (8) evaluated at $f = \frac{1}{2T}$, half the symbol rate, gives

$$\left|P\left(\frac{1}{2T}\right)\right|^2 + \left|P\left(-\frac{1}{2T}\right)\right|^2 = |P(0)|^2.$$

If pulse $p(t)$ is real, as is most often (at least intended to be) the case, then $P(f)$ is conjugate symmetric and $|P(1/2T)| = |P(-1/2T)|$, so the above reduces to

$$\left|P\left(\frac{1}{2T}\right)\right| = \frac{|P(0)|}{\sqrt{2}}. \tag{10}$$

A bandlimited, real, root-Nyquist pulse has a spectrum that is down 3 dB from center at half the symbol rate.

B Design Strategy

System Structure In Fig. 2 the two filters of Fig. 1 are each further partitioned into FIR and nonFIR (usually analog) portions. Each nonFIR portion may in turn comprise several filters. This representation is quite general—any particular filter in the nonFIR portion might be baseband or (the baseband equivalent of) passband, and it might be analog or digital. A digital filter in the nonFIR portion might be IIR or FIR (ignore the terminological difficulty) and such filters might in some cases even operate at different sample rates from the FIR portion. In the nonFIR portion of the transmit filter, the first component "filter" is generally the frequency response (a sinc function of frequency) associated with the sample-to-sample hold of the D/A.

In the development below, it is convenient to permit FIR-filter impulse response ϕ to be written, for example, either as discrete-time sequence ϕ_n or as continuous-time function $\phi(t) = \sum_n \phi_n \delta(t - nT_\phi)$, where T_ϕ is some appropriate sampling interval.

Root-Nyquist Design How can the transmit filter be designed to be root-Nyquist, that is, so that the combination of the transmit filter with its match is Nyquist? First, suppose the nonFIR portion $u(t)$ has already been designed from general system considerations. For the present transmitter-design purposes, assume $v(t) = u^*(-t)$ and $\psi(t) = \phi^*(-t)$, so that the receiver is matched,[3] and system pulse $p \circledast h = \phi \circledast u \circledast v \circledast \psi$ is $(p \circledast h)(t) = \phi(t) \circledast u(t) \circledast u^*(-t) \circledast \phi^*(-t)$. Since for any t this function is linear in the weights of FIR "cascade" filter $g(t) \triangleq \phi(t) \circledast \phi^*(-t)$, linear programming can be used to design $g(t)$ while constraining $(p \circledast h)(t) = g(t) \circledast u(t) \circledast u^*(-t)$ to meet (or approximately meet) the Nyquist zero-ISI requirement $(p \circledast h)(kT) = 0, \forall k \neq 0$, following which g can be spectrally factored[4] into ϕ and its match $\phi^*(-t)$. Samueli [5] pioneered this spectral-factorization approach for the special case in which there is no nonFIR filter, and he showed that it permits constraint of the transmitted spectrum to meet a spectral mask.

The spectral factorization is simple in concept. Transforming $g_n = \phi_n \circledast \phi^*_{-n}$ into the z domain produces $G_z(z) = \Phi_z(z)\Phi_z^*(1/z^*)$, so a zero z_0 of $G_z(z)$ is necessarily accompanied by a "matching" zero at $1/z_0^*$. These matching zeros are at the same angle and reciprocal radii in the complex plane. Enforcing

$$G(f) \geq 0 \tag{11}$$

during optimization causes the zeros to occur in such reciprocal-radii pairs [1, 5]. The subsequent spectal factorization should then simply place one zero of each reciprocal-radii pair into $\Phi_z(z)$ and the other into its match $\Phi_z^*(1/z^*)$. Approaches to the factoring are discussed in [1, 2, 5, 6].

Partitioning Between FIR and nonFIR Filters Knowing the nonFIR portion $u(t)$ of the transmit filter before the the FIR portion $\phi(t)$ is designed not only makes the design of ϕ simpler, it accords with the natural partitioning of duties between the FIR and nonFIR filters. The typical transmit pulse spectrum $P(f)$ is a lowpass function with a much-suppressed stopband. In most applications, the nonFIR filters will be all or mostly analog. Relatively simple analog filters can easily provide large amounts of stopband suppression, but generally only at the expense of a little passband amplitude distortion and perhaps substantial passband phase distortion. A similar statement holds for IIR digital filters. In contrast, small FIR filters can easily be designed to correct such passband distortion, but many taps would be required to provide major stopband suppression. The natural division of labor then is this: the nonFIR portion $U(f)$, which will probably be mostly

[3] Techniques for actual receive-filter design that are compatible with the transmitter-design techniques presented here are discussed in [1, 2, 4].

[4] Only the ϕ factor would be kept; the match ϕ^*_{-n} would be useful only in combination with filter $u^*(-t)$, which is generally impractical to implement.

analog, but which may in some systems contain an IIR component as well, can be designed first to provide most of the large stopband suppression required for $P(f)$. A small FIR portion $\phi(t)$ can then be designed [2,7] to mold both amplitude and phase responses of the passband of the combined $P(f) = \Phi(f)U(f)$ into the desired shapes. The FIR portion may be required to contribute a little stopband suppression as well. Holding FIR stopband suppression to a minimum will result in fewer problems with quantization of filter coefficients.

C Design Example

In Example 1 below, no attempt is made to flatten the eye. Instead, the optimization maximizes the decibel margin by which the spectral-mask specification is met. In this particular design, this will result in somewhat poor use of the mask bandwith, as far as eye flatness is concerned, and so will later serve as an interesting basis for performance comparison when the design is modified to flatten the eye. Also, the present example presents in one place a complete[5] FF pulse-design program, including those details that, though important to a reader considering using FF to do real work, would be a distraction later when eye-flattening techniques are discussed.

Example 1 This transmitter design is for $T^{-1} = 25$ MHz signaling through a what the FCC calls a "40 MHz microwave point-to-point channel" using a real FIR filter operating at $M = 2$ samples per baud. The FF optimization program is presented here incrementally, with discussion of each section.

Defining the nonFIR Portion: The FF begins with this definition of the nonFIR portion $U(f)$ of the transmit filter:

$$\text{Let MHz} = 1\text{, and let } T = \frac{1}{25\,\text{MHz}}.$$

$$\text{Let } M = 2\text{, and}$$
$$\text{define } D(f) \triangleq \frac{\sin(\pi f T/M)}{\pi f T/M}.$$

$$\text{Define } B(s) \triangleq s^3 + 2s^2 + 2s + 1\text{, and}$$
$$\text{define } U(f) \triangleq \frac{D(f)}{B\left(i\frac{f}{25\,\text{MHz}}\right) B\left(i\frac{f}{10\,\text{MHz}}\right)}.$$

The D/A response $D(f)$ is scaled for convenience so that $D(f) \to 1$ as $f \to 0$. (Care will be taken never to actually evaluate $D(0)$, which would result in a divide-by-zero error.) The D/A response shown corresponds to a noncausal impulse response, but that is of no consequence here.

The remainder of the nonFIR filtering comprises two third-order analog Butterworth filters: a 10 MHz baseband "reconstruction" filter and a 25 MHz filter representing a post-conversion RF filter. Figure 3 shows the magnitude responses of the D/A and this dual-Butterworth filter. The RF-filter cutoff is high to keep in-band effects small; larger effects would expose the signal to filter-tuning and component-drift errors that might not be predictable enough to cancel easily with the FIR filter. Those problems may be a little milder at baseband, so the reconstruction-filter cutoff is lower in order to obtain more suppression of the 50 MHz response lobe of the FIR filter. This cutoff is low enough to have a pronounced affect on the signal, but it is not so low as to require huge compensating gains in the FIR portion to make up in-band losses. There is no special reason that both filters are 3*rd*-order Butterworths—it is simply convenient.

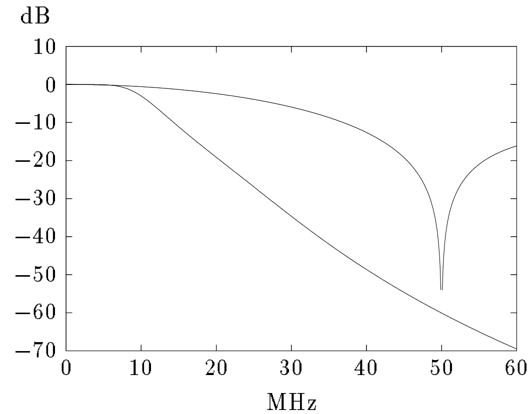


Figure 3: Magnitude responses of the D/A and the dual-Butterworth filter.

The FIR Cascade g_n: Formally define g_n to be a real, even sequence of $2N-1$ elements:

> Let $N = 8$.
> Index g from 0 to $N-1$,
> let g_0 be an optimized, real tap
> at 0 delay, and
> let $g_1, \ldots, g_{N-1}$ be optimized,
> real tap pairs
> at $\pm\frac{T}{M}, \ldots, \pm(N-1)\frac{T}{M}$ delay.
> Let (g, G) be a Fourier transform pair.

Auxiliary optimization variables: FF requires that all optimization variables be declared before any "require..." phrases are used (see below). The array δ of auxilliary optimization variables declared here will be used below in controlling ISI. The scalar ρ will later

[5] except for the spectral factorization

become the factor by which the FCC spectral mask is met.

Let $L = 13$, and index δ from 1 to L.
Let $\delta_1, \ldots, \delta_L$ be optimized
and bounded below by 0.
Let ρ be optimized
and bounded below by 0.

Factorability of the Cascade: If g_n is to be eventually factored as $\phi_n \circledast \phi^*_{-n}$, the requirement (11) of a nonnegative frequency response $G(f)$ must be met.

$\forall f \in \left(0, \frac{M}{2T}\right]$ with $\Delta \leq 0.1$ MHz,
require $G(f) \geq 0$.

The "$\forall f \in SomeSet$" notation indicates that the set $SomeSet$ (here a half-open interval) is to be sampled with a sample spacing not exceeding the bound on Δ that follows. The "require..." phrase applies at each frequency sample. Since g_n is real and even, so is $G(f)$, hence constraints for negative f are not needed.

Intersymbol Interference: By (5), normalized peak ISI (worst-case eye closure) is just $\sum_{m \neq 0} |q_m|$, where $q_n = \kappa^{-1}(p \circledast h)(\tau + nT) = \kappa^{-1}[g(t) \circledast u(t) \circledast u^*(-t)]_{t=nT}$. In the FF below, approximate inverse Fourier transformation of even function $|P(f)|^2 = G(f)|U(f)|^2$ represents the two convolutions; the product $k\,df$ represents frequency in the Fourier sum. System gain $\kappa = (p \circledast h)(\tau)$ is set according to (7) to cause $|P(0)| \approx 1$. For later convenience, the function q is made a function of continuous time in the FF, so that $q(nT)$ in the FF is equivalent to q_n in the mathematical development above.

Let $\kappa = 1/T$.

Let $df = \frac{1}{250T}$, and
define $q(t) \triangleq \dfrac{df}{\kappa}\left[G(0) + 2\sum_{k=1}^{600} G(k\,df)|U(k\,df)|^2 \cos(2\pi k\,df\,t)\right]$.

Require $q(0) = 1$, and
for each $n = 1, \ldots, L$,
require $|q(nT)| \leq \delta_n$.
Require $2\sum_{n=1}^{L} \delta_n \leq 0.005$.

The first term in the square brackets should be $G(0)|U(0)|^2$, but evaluating $U(0)$ evaluates $D(0)$ which results in division by zero; omitting the $|U(0)|^2$ factor is harmless here, because it equals unity. Since $q(t)$ is even, constraints for negative n are unnecessary.

The final two lines of FF above give

$$2\sum_{n=1}^{L} |q(nT)| \leq 2\sum_{n=1}^{L} \delta_n \leq 0.005,$$

effectively limiting the normalized peak ISI of (5) to 0.5%.[6]

The Spectral Mask: A mask to constrain the transmitted spectrum is typically[7] specified by regulation as a bound at each frequency f on the fraction of the transmitted power that can fall in a small frequency interval of some specified width ϵ centered at f. The FF below defines $S(f)$ to be the transmitted spectrum (6), assuming for convenience that $\sigma_a^2 = 1$. The function $\Lambda(f)$ then represents the relative power of regulatory interest. The denominator is the total transmitted power κ/T from (9). The rest of the FF constrains $\Lambda(f)$ according to a mask[8] for a microwave digital-radio channel of nominal bandwidth W that falls from -50 dB at frequency α to -80 dB at frequency β and then levels off. This mask will just be met if $\delta = 1$.

Define $S(f) \triangleq \dfrac{1}{T}G(f)|U(f)|^2$.
Let $\epsilon = 0.004$ MHz, and
define $\Lambda(f) \triangleq \dfrac{S(f)\,\epsilon}{\kappa/T}$.
Let $W = 40$ MHz,
let $\alpha = W/2$, and
let $\beta = W[17 - 2\log_{10}\left(\frac{W}{1\,\mathrm{MHz}}\right)]/16$.

Let $\gamma = 10^4$.
$\forall f \in [\alpha, \beta]$ with $\Delta \leq 1$ MHz,
require
$\gamma\Lambda(f) \leq \gamma 10^{-2/10} 10^{-\frac{80(f-\alpha)+50(\beta-f)}{10(\beta-\alpha)}}$;
$\forall f \in (\beta, 2.5\,W]$ with $\Delta \leq 1$ MHz,
require $\gamma\Lambda(f) \leq \gamma 10^{-2/10} 10^{-80/10}$.

Without the the seemingly irrelevant scale factor of γ on both sides of the inequalities, the coefficient of ρ is only 10^{-8} for $f \geq \beta$. The small coefficients ultimately lead to numerical problems in the simplex algorithm used by the FF processor to solve the linear program. These problems result in an incorrect report of "conflicting constraints."

[6] It would of course have been easier to simply require $q(n) = 0$ for some suitable range of n, but each such equality constraint consumes a degree of optimization freedom, so one would expect the present less-severe specification to result in better optimization performance. Design examples in [3] suggested empirically that minimizing normalized peak ISI has the effect of setting $q(nT) = 0$ for some range of n, but with that range chosen optimally.

[7] Sometimes a limitation on total out-of-band power is specified instead. The control of such out-of-band power in the linear program, as an alternative to the spectral-mask strategy presented here, is straightforward.

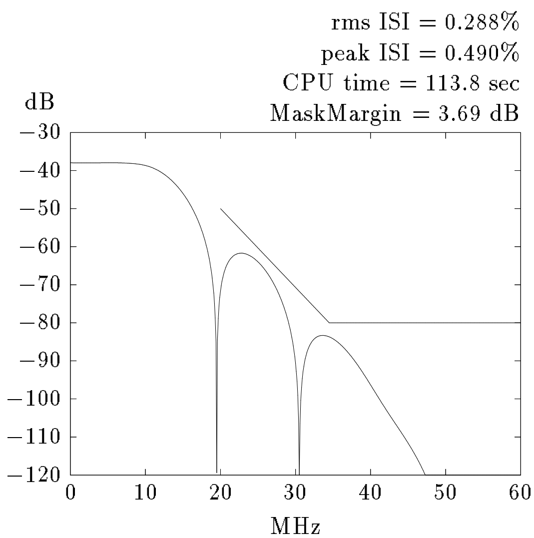
[8] FCC rule §21.106(a)(2)(i)

Optimization and Output: The optimization maximizes and outputs the margin by which the spectral mask is met.

Minimize ρ,
write $-10\log_{10}(\rho)$ to "MaskMargin", and
for each $n = 0, \ldots, N-1$,
  write g_n to "Weights".

This completes the FF.

The "peak ISI" shown with the plot below of the optimized spectrum is given as a percentage of the peak symbol value and is calculated from (5); likewise, "rms ISI" [1] is given as a percentage of rms symbol amplitude.



The fractional spectral power $\Lambda(f)$ and its regulatory mask.

Although the phase of $\Phi(f)$ itself cannot be determined without first factoring $G_z(z)$ to obtain $\Phi_z(z)$, the magnitude response $|\Phi(f)|$ can already be computed as $|G(f)|^{1/2}$ (see below). The mask ensures a sharp rolloff of the transmit spectrum $S(f)$ with frequency, making it more-or-less bandlimited. The ISI constraints then force the spectrum (and hence $P(f)$) to be practically flat across most of the band. To obtain this spectrum the FIR portion of the filter must have a slightly peaked response to make up for the droop in the response of the nonFIR portion.[9] In this particular design, essentially all of the required stopband suppression comes from the nonFIR portion of the filter.

[9] There is one special frequency: At $f = 1/2T$ the value of $|\Phi(f)|$ is completely determined by $U(f)$ through (10). The optimization is not free to control $|\Phi(f)|$ at that frequency.

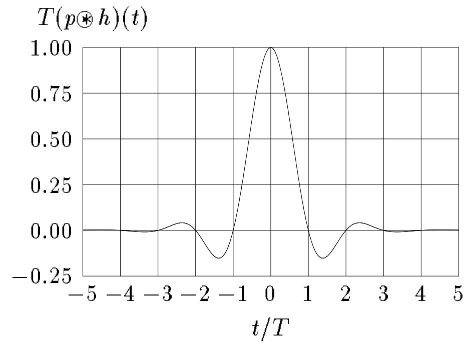
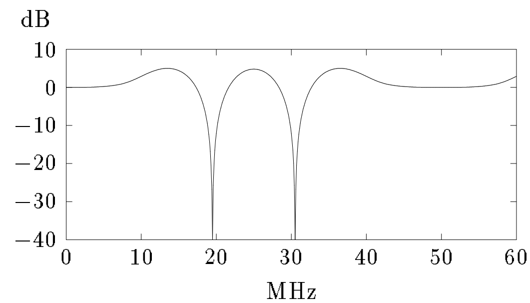


Figure 4: The system pulse produced by a matched receiver in Example 1.



Magnitude response $20\log|\Phi(f)|$ of the FIR portion of the transmit filter.

Fig. 4 shows the matched-receiver system pulse, and the upper-left part of Fig. 5 shows the corresponding eye diagram (discussed below). [1]

III FLATTENING THE EYE

A Theory

Timing Jitter Attainment of the low ISI levels theoretically associated with the system pulses of recent examples depends critically on correct receiver sample timing. The Example 1 system pulse in Fig. 4, for example, was optimized for a $\tau = 0$ value for the timing variable in the matched receiver of Fig. 1. As τ deviates from this optimum, the peak-ISI terms (from (5)) of

$$\frac{\text{largest ISI}}{\text{largest desired signal}} = \sum_m |q_m - \delta_m| = \frac{1}{(p\circledast h)(\tau)} \sum_{m\neq 0} |(p\circledast h)(\tau + mT)|$$

increase rapidly. While examination of the system pulse makes this qualitatively clear, an eye diagram provides a better intuitive feel for the degree of ISI increase.

The upper-left part of Fig. 5 shows an *eye* diagram for Example 1, so named after the central, diamond-shaped opening. The diagram extends horizontally in sampling time τ from $-T/2$ to $T/2$, and the darkness versus vertical position at time τ shows the probability density[10] of

$$r_k = \sum_m a_m (p \circledast h)(\tau + (k-m)T),$$

a zero-noise version of (1), under the assumption that the symbols $\{a_m\}$ are drawn independently and equiprobably from a two-element constellation. Alternatively, one might view the diagram as an oscilloscope image of

$$\left(\sum_m a_m p(t - mT)\right) \circledast h(t),$$

the output of the receive filter of Fig. 1 with random binary symbols but without noise, with the oscilloscope triggered once per baud at a point one half baud before the optimal sampling time. Either interpretation of the eye diagram reveals clearly the peak-ISI increase as timing deviates from the $\tau = 0$ sampling point.

Optimization Strategies To make an eye like the left eye in Fig. 5 more tolerant of timing error, reshape the eye opening so that the edges slope less steeply approaching the zero-ISI sample point. (Such reshaping will be visibly significant only if the spectral-mask requirement permits adequate control of the pulse shape.)

One possible approach to improving tolerance to timing error is to maximize the height of the eye opening along a vertical cut some fraction of a baud, say one tenth, from the center while bounding peak ISI.

A similar but slightly more elegant approach to improving the eye results from using the derivative of the system pulse to get at the slope of the eye opening. Since that slope is given by the maximum magnitude of $\sum_m a_m (p \circledast h)'(\tau + (k-m)T)$, where prime denotes the derivative, an argument parallel to that used to derive (5) gives this maximum magnitude as $a \sum_m |(p \circledast h)'(\tau + mT)|$. Normalization by the maximum desired signal, as was done for peak ISI, results in

$$\frac{\text{largest slope}}{\text{largest desired sig}} = \sum_m |\kappa^{-1}(p \circledast h)'(\tau + mT)|.$$

Example designs illustrating both approaches are carried out below.

Multipath To represent a digital-radio system accurately, the the simple channel model in Fig. 1 generally must be modified—the transmitter output $s(t)$ must be convolved with some $c(t)$ prior to the addition of the noise. The complex channel impulse response $c(t)$ models the effect of radio propagation over multiple signal paths, each with its own delay, gain, and phase shift. If $c(t)$ were known exactly, the FF design of the transmit filter might compensate for it by including $|C(f)|^2$ (which includes the matched filter) in the ISI-controlled system pulse (but not in the spectral-mask requirement). Usually, however, only general properties of $c(t)$ are known; the most important of these characteristics is its duration. Generally, the shorter the duration, the better.

Suppose $c(t)$ is nonzero only in a short time interval $[-\tau_c, \tau_c]$, and suppose τ_c is sufficiently small that system pulse $f(t)$ can be approximated as linear in $[\tau + kT - \tau_c, \tau + kT + \tau_c]$, for each k. Then the effect of the channel on the system pulse at the times $\tau + kT$ relevant to ISI is given by

$$\begin{aligned} &|(f \circledast c)(\tau + kT)| \\ &\quad = \left| \int c(u) f(\tau + kT - u)\, du \right| \\ &\quad \approx \left| \int c(u)[f(\tau + kT) - f'(\tau + kT)u]\, du \right| \\ &\quad \le |f(\tau + kT)| \left| \int c(u)\, du \right| \\ &\qquad + |f'(\tau + kT)| \left| \int c(u) u\, du \right|. \end{aligned}$$

It is now clear that minimization of the contribution of the multipath-corrupted system pulse to ISI requires minimization at the ISI-sample times $\{\tau + kT\}$ of the magnitudes of both the system pulse and its derivative. Thus, the techniques proposed above for dealing with timing jitter are directly applicable.

[10] For some symbol constellations and system pulses, r_k can have a *singular* distribution function [8], one that is continuous everywhere and has a zero derivative almost everywhere [9], so causing the associated probability density to be zero and thereby failing to usefully represent the distribution. When such a case arises, a nonzero density that integrates to an approximate distribution function offers a practical way out.

B Design Examples

In Example 1, the optimization objective was maximization of the decibel margin by which the spectral mask was met. If a bound on that margin is used instead, then a new objective can be fashioned to flatten the eye in one of the ways discussed above. If the 3.69 dB by which the mask was met in Example 1 were used as this new bound and no other constraints or parameters were changed, we would expect no improvement in the newly optimized quantity, whatever that quantity might be, over the level realized in Example 1, because we would have created a problem with just a single feasible solution. To improve our new objective, whatever it may be, we must either ease up on the spectral-mask requirement or change some other aspect of the system. The new objective here will be some measure of eye flatness, so the amount of improvement expected will be a function of those changes. In the following examples, I arbitrarily chose to ease the spectral-mask margin to 2 dB and to increase the number of taps in the transmit filter from $N = 8$ to $N = 12$.

Example 2 This example attempts to improve tolerance to timing error by maximizing the height of the eye opening along a vertical cut one tenth baud from the center while bounding peak ISI. The FF differs from Example 1 in several ways:

Number of Taps: The transmit filter has $N = 12$ taps.

Auxiliary optimization variables: This section includes some new variables:

Let $L = 8$.

Index δ from 1 to L, and
let $\delta_1, \ldots, \delta_L$ be optimized
and bounded below by 0.

Index ρ from $-L$ to L, and
let $\rho_{-L}, \ldots, \rho_L$ be optimized
and bounded below by 0.

Intersymbol Interference: This section is as before with the addition of the following:

For each $n \in \bigcup_{k=1}^{L} \{-k, k\}$,
require $|q(0.1T + nT)| \leq \rho_n$;
require $\sum_{n=1}^{L} (\rho_n + \rho_{-n}) \leq \rho_0$.

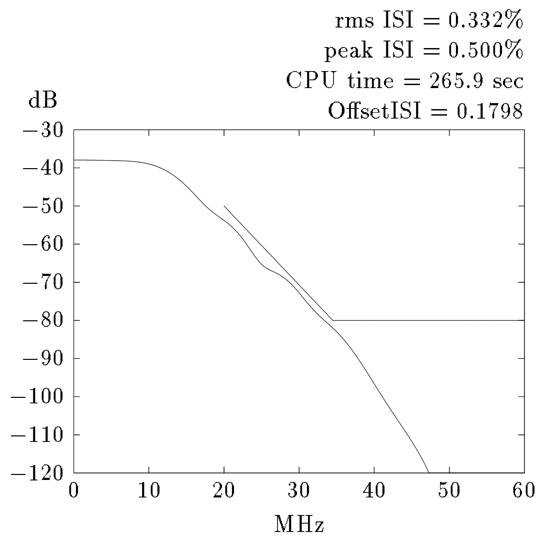
The sum in the inequality $\sum_{n \neq 0} |q(0.1T + nT)| \leq \rho_0$ is proportional to peak ISI when the timing error is $0.1T$.

Do not mistake $\{-k, k\} \equiv \{-k\} \cup \{k\}$ for an interval like $(-k, k)$ or $[-k, k]$.

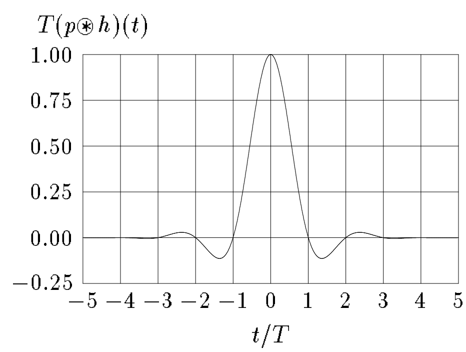
The Spectral Mask: Here ρ was changed to $10^{-2/10}$ so that the margin by which the mask is met is not optimized but simply limited to 2 dB.

Optimization and Output: Minimize peak ISI with a timing error:

Minimize ρ_0, write ρ_0 to "offsetISI", and
for each $n = 0, \ldots, N-1$,
write g_n to "Weights".



The fractional spectral power $\Lambda(f)$ and its regulatory mask.



The system pulse produced by a matched receiver.

The middle eye in Fig. 5 is indeed slightly more "open" for offset timing than is the left eye corresponding to Example 1. 2

The next example demonstrates the minimization of the slope of the eye opening.

Example $\boxed{3}$ This example is like Example 2 except for the following sections:

Intersymbol Interference:
This section is as before but with Fourier-sum function $\xi(t)$ added below to approximate $-\kappa^{-1}\int 2\pi fT|P(f)|^2 \sin 2\pi ft\, df$, which is the inverse Fourier transform of the imaginary, odd function $2\pi ifT\kappa^{-1}|P(f)|^2$, which in turn is just the Fourier transform of the derivative of $\kappa^{-1}(p \circledast h)(t)$ with respect to normalized time t/T.

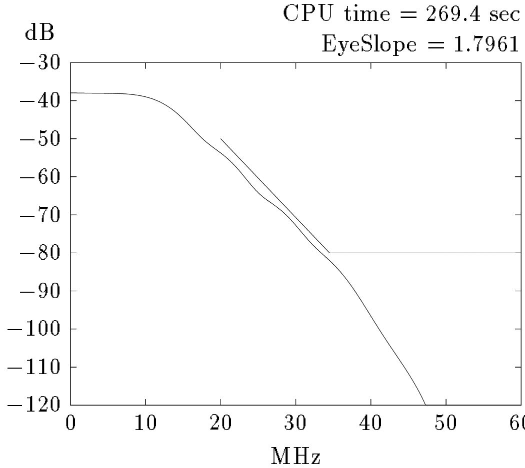
Define $\xi(t) \triangleq$
$$-\frac{2}{\kappa}\sum_{k=1}^{600} 2\pi k\, df\, TG(k\, df)|U(k\, df)|^2 \times \sin(2\pi k\, df\, t)\, df.$$

For each $n = -L, \ldots, L$,
require $|\xi(nT)| \le \rho_n$.

Optimization and Output: The constraints at the end of the ISI section above, together with minimization of $\sum_n \rho_n$ below, ensure that $\sum_n \rho_n$ in fact represents the eye slope:

Minimize $\sum_{n=-L}^{L} \rho_n$,
write $\sum_{n=-L}^{L} \rho_n$ to "EyeSlope", and
for each $n = 0, \ldots, N-1$,
write g_n to "Weights".

rms ISI = 0.351%
peak ISI = 0.500%
CPU time = 269.4 sec
EyeSlope = 1.7961



The fractional spectral power $\Lambda(f)$ and its regulatory mask.

The spectral plot above is very similar to that of Example 2, and the system pulse here was nearly indistinguishable visually from that of Example 2 and so is not shown, but a comparisons of the middle and right eye diagrams in Fig. 5 reveals differences. $\boxed{3}$

The numerically computed eye-opening slope, shown below each of the lower eyes in Fig. 5, shows a modest improvement from Example 1 to Example 2, an improvement commensurate with the limited excess bandwidth available under this spectral mask, but from Example 2 to Example 3 there is practically no change. Performance is nearly identical.

IV SUMMARY AND CONCLUSIONS

This paper presents complete and working FF optimization programs for example designs demonstrating two strategies for making the eye opening flatter near the optimal sampling instant in an hybrid analog/FIR system for shaping the transmit pulse. In the optimization, peak ISI is constrained to a specified bound, and the transmitted spectrum is constrained to fall under a spectral mask. It is the FIR portion of the hybrid system that is optimized, resulting automatically and naturally in "compromise" equalization of the distortion caused by the already-designed analog portion of the transmit filter. Because the FIR design process will compensate for the analog filter in this way, the analog filter can be designed to begin its rolloff early, thereby requiring a lower-order analog filter. Little rolloff is required of the FIR filter, so its order is small also.

The two optimization strategies for flattening the eye are (1) directly maximize the eye opening at an offset sampling epoch, or (2) directly minimize the slope of the eye-opening edge adjacent to the optimal sampling point by minimizing the sum of the absolute values of T-spaced samples of the derivative of the system pulse. Although the latter is more mathematically appealing, the two techniques appear to perform equally well.

The demonstration of the design power and flexibility that results from the use of a special-purpose optimization language is perhaps just as important. The **ffc** interpreter for the FF language is currently available by anonymous ftp from **ftp.ee.mtu.edu** in directory **/pub/jeffc**.

REFERENCES

[1] Jeffrey O. Coleman. *The Use of the FF Design Language for the Linear-Programming Design of Finite-Impulse-Response Digital Filters for Digital-Communication and Other Applications.* PhD thesis, University of Washington, Seattle, WA, December 1991.

[2] Jeffrey O. Coleman and Dean W. Lytle. "A Linear-Programming Design Language and its Application to Data-Transmission Filters". In *Conference Record, Pacific Rim Conference on Communications, Computers, and Signal Processing*, pages 778–782. IEEE, May 1991.

[3] Jeffrey O. Coleman and Dean W. Lytle. "Linear-Programming Techniques for the Control of Intersymbol Interference with Hybrid FIR/Analog Pulse Shaping". In *Conference Record, International Conference on Communications*, pages 329.2.1–329.2.6. IEEE, June 1992.

[4] Jeffrey O. Coleman. "Linear-Programming Design of Hybrid Analog/FIR Receive Filters to Minimize Worst-Case Adjacent-Channel Interference". In *Proceedings, Conference on Information Sciences and Systems*, March 1993.

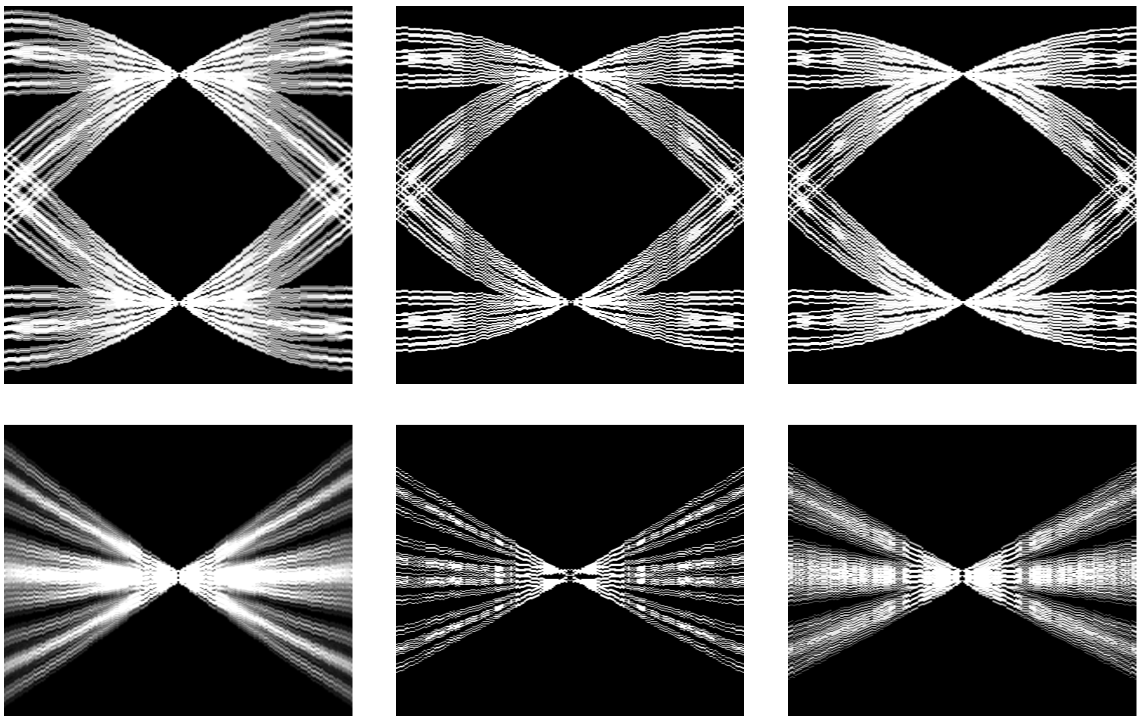
[5] Henry Samueli. "Linear Programming Design of Digital Data Transmission Filters with Arbitrary Magnitude Specifications". In *Conference Record, International Conference on Communications*, pages 30.6.1–30.6.5. IEEE, June 1988.

[6] G. A. Mian and A. P. Nainer. "A Fast Procedure to Design Equiripple Minimum-Phase FIR Filters". *IEEE Transactions on Circuits and Systems*, CAS-29(5):327–331, May 1982.

[7] P. Vandamme. "On the Synthesis of Digital Transmit Filters". *IEEE Transactions on Communications*, 39(4):485–487, April 1991.

[8] Paul H. Wittke, Wendy S. Smith, and L. Lorne Campbell. "Infinite Series of Interference Variables with Cantor-Type Distributions". *IEEE Transactions on Information Theory*, 34(6):1428–1436, November 1988.

[9] Patrick Billingsley. *Probability and Measure.* Wiley, New York, second edition, 1986.



2.2898 1.8157 1.8106

Figure 5: Eye diagrams and $10\times$ closeups for Examples 1 (left), Examples 2 (middle), and 3 (right). The numbers are estimates of $\frac{\text{largest slope}}{\text{largest desired signal}} = \sum_m |\kappa^{-1}(p \circledast h)'(\tau + mT)|$ computed numerically from the system pulse.