LINEAR-PROGRAMMING DESIGN OF HYBRID ANALOG/FIR RECEIVE FILTERS TO MINIMIZE WORST-CASE ADJACENT-CHANNEL INTERFERENCE

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ABSTRACT This paper explores several linear-programming techniques for the design of the FIR part of an analog/FIR hybrid receive filter with the goal of minimizing adjacent-channel interference (ACI) in a high-speed digital-radio modem. Four methods are presented: (1) minimizing peak ACI spectral height, (2) minimizing the L_1 norm of the ACI spectrum, (3) minimizing peak time-domain ACI-pulse height, and (4) minimizing worst-case time-domain ACI, approximated as the sum of the complex magnitudes of baud-spaced samples of the ACI pulse with the worst-case adjacent-channel timing epoch. The second and fourth methods give excellent performance.

I. INTRODUCTION

A high-rate digital-communication modem requires specialized filters in its signal path, and finite-impulse-response (FIR) digital filters provide the design flexibility that allows the needed characteristics to be economically realized. The system designer can exploit the inevitable presence of associated analog filtering to keep the required number of FIR taps small by designing the analog filter(s) to share some of the spectral-shaping burden with the FIR filter, thereby creating a true hybrid analog/FIR design [1, 2]. Linear-programming approaches to the design of transmit filters based on this idea have been explored both for microwave digital radio [1–3] and for optimal sub-Nyquist signaling [4, 5], but the flexibility that linear programming brings to the design of modem receive filters appears heretofore unexamined, except in the most preliminary way [1]. Through design examples, this paper explores several linear-programming techniques for the design of the FIR component of such an analog/FIR hybrid receive filter with the goal of minimizing the effects of adjacent-channel interference (ACI).

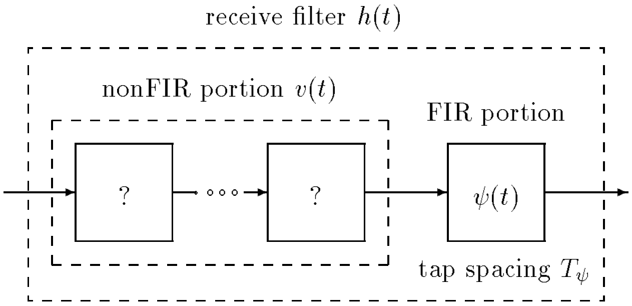


Figure 1: The receive filter $h(t)$.

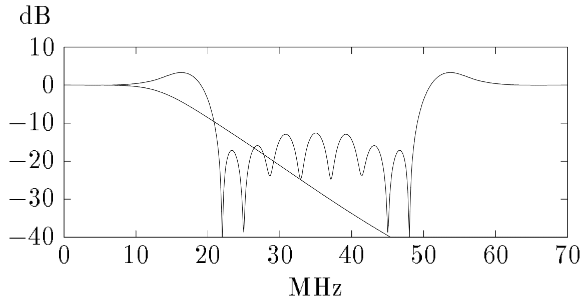


Figure 2: Responses $|V(f)|$ and $|\Psi(f)|$ of Example 2.

Assume a known transmit pulse $p(t)$ is to be passed through an AWGN channel into a receive filter with impulse response $h(t)$ (to be designed), whose output will be sampled at sampling epoch τ. In Fig. 1 the receive filter $h(t)$ is shown partitioned into a to-be-designed FIR portion $\psi(t)$ and a known nonFIR portion $v(t)$. (The latter may comprise several filters.) Knowing the nonFIR portion $v(t)$ of the receive filter before the the FIR portion $\psi(t)$ is designed not only makes the design of ψ simpler, it accords with the natural partitioning of duties between the FIR and nonFIR filters. The typical receive-filter frequency response $H(f)$ is a lowpass function with a much-suppressed stopband. In most applications, the nonFIR filter will be all or mostly analog. Relatively simple analog (or IIR digital) filters can easily provide large amounts of stopband suppression, but generally only at the expense of some passband amplitude and phase distortion. In contrast, small FIR filters can easily be designed to correct such passband distortion, but many taps would be required to provide major stopband suppression. The natural division of labor then is this: the nonFIR portion $V(f)$ should be designed first (here assumed) to provide most of the large stopband suppression required for $H(f)$. A small FIR portion $\Psi(f)$ can then be designed (the present task) to mold the passband response of the combined $H(f) = \Psi(f)V(f)$ into the desired shape while contributing any remaining minor stopband suppression required,[1] as illustrated in Fig. 2, where the FIR filter contributes only 15 dB or so of the overall stopband rejection.

II. DESIGN EXAMPLES

A. Minimizing Frequency-Domain ACI

In the first two examples ACI is minimized in the frequency domain through direct control of the "ACI spectrum," the response of the receive filter to a pulse transmitted in an adjacent channel.

The Frequency-Domain ACI L_∞ Norm The first example develops a reasonably complete design based on a straightforward approach to ACI control. ACI is controlled by minimizing the L_∞ norm (here the least upper bound of the complex magnitude) of the ACI spectrum. This simple example provides a performance baseline against which the more-sophisticated design techniques of the later examples can be compared. Actual design code is given in the self-documenting FF linear-programming language first presented in [1] and fully documented in [2].

Example $\boxed{1}$

This design is for a 35 Msymbols/s digital-radio microwave link operating on a 40 MHz FCC point-to-point microwave channel. The known transmit pulse, itself designed using linear programming, is the minimum peak-power version [2, page 113] of the Example 5 pulse of [3] (the Example 4-5 pulse of [2]).

Defining the Fixed Transmit Pulse: The FF begins with this formal definition of the (transform

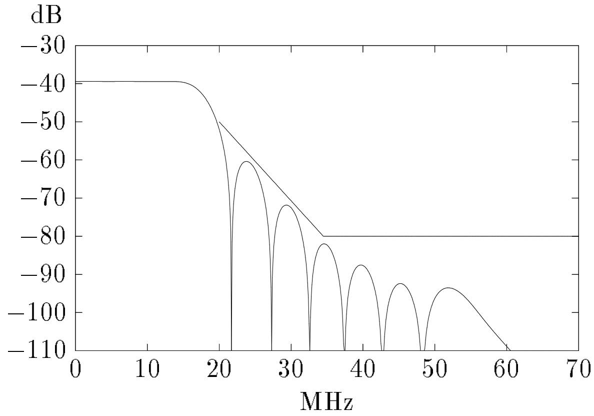


Figure 3: The transmitted spectrum, normalized for comparison against the regulatory mask.

$P(f)$ of the) known transmit pulse $p(t)$:

Let MHz = 1, and let $T = \dfrac{1}{35\ \mathrm{MHz}}$.

Let $N = 15$, and let $M = 2$.
Interpolate phi from real file "TxTaps"
with spacing 1 from 0.
Index ϕ from 0 to $N-1$,
let $\phi_0, \ldots, \phi_{N-1}$ be fixed, real taps
at $0, \ldots, (N-1)\frac{T}{M}$ delay, and
for each $k = 0, \ldots, N-1$, let $\phi_k = \mathrm{phi}(k)$.
Let (ϕ, Φ) be a Fourier transform pair.

Define $D(f) \triangleq \dfrac{\sin(\pi f T/M)}{\pi f T/M} e^{-i\pi f T/M}$,
define $B(s) \triangleq s^3 + 2s^2 + 2s + 1$, and
define $U(f) \triangleq \dfrac{D(f)}{B\left(i\frac{f}{30\ \mathrm{MHz}}\right) B\left(i\frac{f}{15\ \mathrm{MHz}}\right)}$.

Define $P(f) \triangleq \Phi(f)U(f)$.

This particular transmit filter comprises two analog 3*rd*-order Butterworth filters, with 15 MHz and 30 MHz cutoffs, in cascade with a D/A $D(f)$ and a 15-tap FIR filter $\Phi(f)$ operating at $M = 2$ samples/symbol. The coefficients for the FIR filter are read from file "TxTaps" with the "Interpolate..." statement. (These unimportant details are included for completeness.) Figures 3 and 4 show the transmit spectrum, FCC mask,[2] and pulse $p(t)$.

Defining the Receive Filter: After defining the FIR portion ψ_n of the receive filter to be real with K taps and to operate at $M = 2$ samples per baud, the FF below defines the nonFIR portion $V(f)$ of the receive filter to be the

[1] This was demonstrated by this author [1, 2] and also suggested independently by Said [6]

[2] FCC rule §21.106(a)(2)(i)

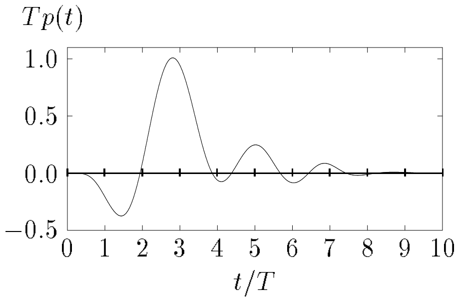


Figure 4: The known transmit pulse $p(t)$ assumed in the receiver design examples.

same (for calculation convenience only) dual-Butterworth used in the transmitter. The 30 MHz filter might be an image-suppression RF or IF filter, and the 15 MHz filter might represent a baseband antialiasing and mixer-product-suppression filter. Figure 2 plots $|V(f)|$ in dB.

Let $K = 20$,
index ψ from 0 to $K-1$, and
let $\psi_0, \ldots, \psi_{K-1}$ be optimized, real taps at $0, \ldots, (K-1)\frac{T}{M}$ delay.
Let (ψ, Ψ) be a Fourier transform pair.

$$\text{Define } V(f) \triangleq \frac{1}{B\left(i\frac{f}{30\text{ MHz}}\right) B\left(i\frac{f}{15\text{ MHz}}\right)}.$$

$$\text{Define } H(f) \triangleq V(f)\Psi(f).$$

Filter ψ_n is not required to have linear phase.

Auxiliary optimization variables: The δ family of auxiliary variables declared here will be used later for intersymbol interference (ISI) control. Variable ρ will be used for control of ACI.

Let $L = 13$, index δ from $-L$ to L, and
for each $n \in \bigcup_{k=1}^{L} \{-k, k\}$,
let δ_n be optimized
and bounded below by 0.

Let ρ be optimized
and bounded below by 0.

Intersymbol Interference: Any of the several transmitter-design ISI-control techniques explored in [3] can be adapted for use here, without regard to the method used to design the transmit pulse. The FF below uses a peak-ISI minimization technique similar in spirit to that of Example 5 of [3].

If $\kappa = (p\circledast h)(\tau)$ denotes system gain, then zero ISI results when $q(n) = 0, \forall n \neq 0$, where $q(n) = \kappa^{-1}(p\circledast h)(\tau + nT)$. In this design, peak ISI is bounded explicitly by limiting $\sum_{n\neq 0}|q(n)|$ to 0.5%.

An approximation to the (real) inverse Fourier transform of system function $P(f)H(f)$ represents the $q(n)$ convolution below. The product $k\,df$ represents frequency in the Fourier sum.

Let $\kappa = \dfrac{|\Phi(0)|^2}{T}$, and let $\tau = 11T$.

Let $df = \frac{1}{1000T}$, and
$$\text{define } q(n) \triangleq \frac{df}{\kappa}\left[\Phi(0)H(0) + 2\sum_{k=1}^{2000} \Re\left(P(k\,df)H(k\,df)e^{2\pi i k\,df(\tau+nT)}\right)\right].$$

Require $q(0) = 1$, and
for each $n \in \bigcup_{k=1}^{L} \{-k, k\}$,
require $|q(n)| \leq \delta_n$.
Require $\sum_{n=1}^{L}(\delta_{-n} + \delta_n) \leq 0.005$.

In two places $\Phi(0)$ is used in place of $P(0) = \Phi(0)U(0) = \Phi(0)$, to avoid division by zero in $D(0)$. System gain κ is set to cause $|H(0)| \approx 1$. Sampling epoch τ is set arbitrarily; any value for τ over a several-baud range can generally be used if there are "plenty" of taps in the FIR portion ψ of the receive filter. With fewer taps, the choice is more critical, and a τ resulting in good optimization performance must be chosen experimentally.

Adjacent-Channel Interference: The heart of this design is a modest effort to control ACI by bounding the magnitude of the ACI spectrum:

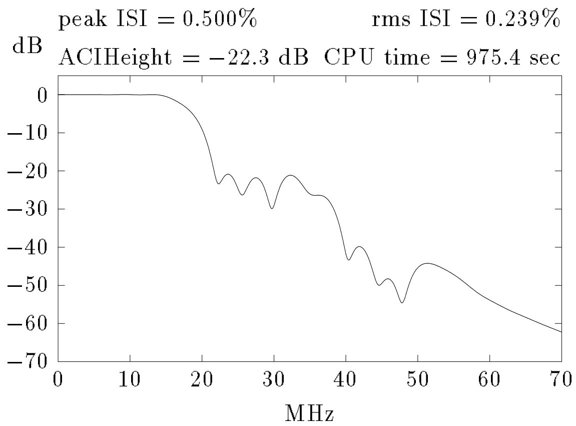
Let $W = 40$ MHz, and
$\forall f \in (0, W)$ with $\Delta \leq 1$ MHz,
require $\|P(f-W)H(f)\|_6 \leq \rho$.

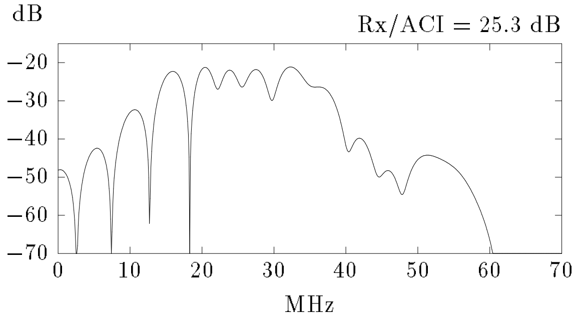
This ACI spectral bound would ideally use the magnitude $|\cdot|$ of the spectrum, but that cannot be accomplished with linear constraints, so a hexagon norm, denoted $\|\cdot\|_6$, is used as an approximation. Just as a complex number has less than unit magnitude if it is inside the unit circle, it has less than unit hexagon norm if it is inside an origin-centered hexagon one of whose sides is bisected by the point 1 on the real axis, a "unit hexagon."

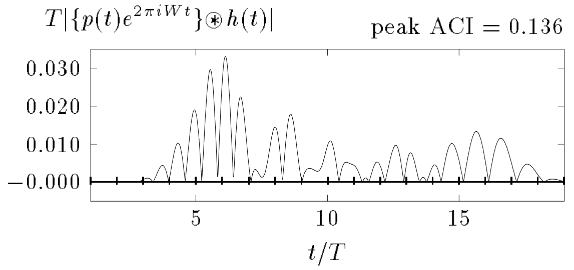
Optimization and Output: To complete the FF, minimize and output peak ACI spectral height:

Minimize ρ, and
write $20\log_{10}(\rho)$ to "ACIHeight".
$\forall n = 0, \ldots, K-1$, write ψ_n to "Weights".

The magnitude response of receive filter $H(f) = \Psi(f)V(f)$ is plotted below, with optimization performance above the plot,[3] and a plot of ACI spectral magnitude follows, where the quantity "Rx/ACI" is the ratio of the receive-filter response energies from transmit pulse $P(f)$ and ACI pulse $P(f-W)$.







Finally, the complex magnitude of the "ACI pulse," the time-domain response of the receive filter to a pulse transmitted in an adjacent channel, appears above. The quantity "peak ACI" will be discussed in conjunction with Example 4 and appears here for comparison. $\boxed{1}$

In Example 1, the sidelobes (say 22 to 48 MHz) of $\Psi(f)$ are so poorly suppressed by the objective function that no spectral nulls appear. The mild dips that appear instead carry over to the spectral-peak region of the ACI spectrum. Why was ρ not smaller? The Nyquist criterion implies that $H(f)$ can be no more than 3 dB below $H(0)$ at $f = 1/2T = 17.5$ MHz (assuming that $P(f)$ is root-Nyquist), so the height of the Example 1 ACI response $H(f)P(f-W)$ is predetermined at $f = 17.5$ MHz by the ISI constraints. The ACI spectral bound ρ cannot be forced much lower, no matter how many taps are added, etc. This is why response $H(f)$ has null-free sidelobes: the "slack" arises because the design has more taps than needed to achieve the bound on ρ.

The Frequency-Domain ACI L_1 Norm In order to further lower the received ACI power then, we must force *part* of the ACI spectrum lower, even though we cannot force it lower uniformly across its width. The next example does so by approximating the L_1 norm of the ACI spectrum.[4]

Example $\boxed{\textbf{2}}$ This design uses the FF of Example 1, except for the following sections:

Auxiliary optimization variables: The δ family of auxiliary variables is as before, but ρ is now an array of optimization variables.

> Let $L = 13$, index δ from $-L$ to L, and
> for each $n \in \bigcup_{k=1}^{L} \{-k, k\}$,
> let δ_n be optimized
> and bounded below by 0.
>
> Index ρ from 1 to 39.
> Let $\rho_1, \ldots, \rho_{39}$ be optimized
> and bounded below by 0.

Adjacent-Channel Interference (ACI): Each of the MHz-spaced ACI spectral-magnitude samples is now bounded by a separate auxiliary variable.

> Let $W = 40$ MHz, and let $df = 1$ MHz.
> For each $k = 1, \ldots, 39$,
> require $\|P(k\,df - W)H(k\,df)\|_6 \le \rho_k$.

Optimization and Output: Here the optimization minimizes and outputs the sum of the ACI bounding variables. Because the sum is minimized, the "$\le$" above in the ACI requirement holds with equality, and the sum approximates $\int |P(f-W)H(f)|\,df$, the L_1 norm of ACI spectrum $P(f-W)H(f)$:

[3] The CPU time shown is for a Sun SPARCstation 2 running a heavily-instrumented, slow version of the FF processor.

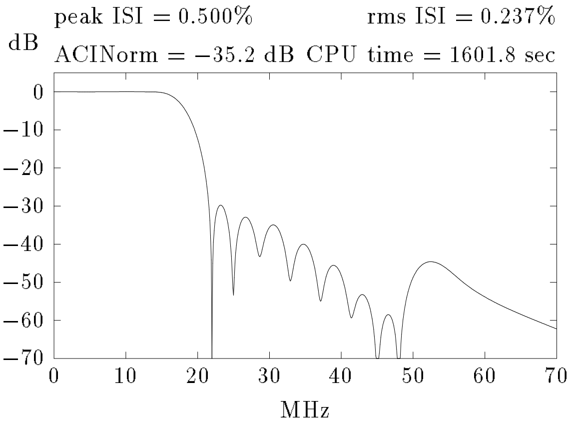
[4] The control of approximate L_1 frequency-response norms in linear-programming FIR filter designs has been reported previously in [2, 7].

$$\text{Minimize } \sum_{k=1}^{39} \rho_k, \text{ and}$$

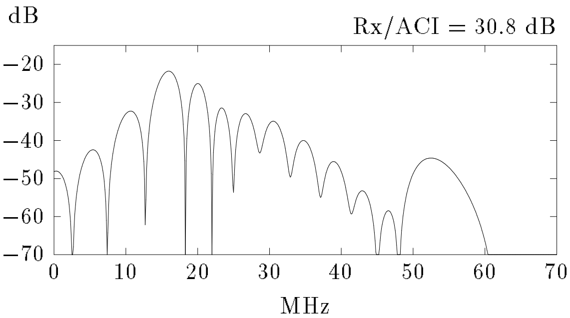
$$\text{write } 20\log_{10}\left(\frac{1}{39}\sum_{k=1}^{39}\rho_k\right) \text{ to "ACINorm".}$$

$$\forall n = 0, \ldots, K-1, \text{ write } \psi_n \text{ to "Weights".}$$

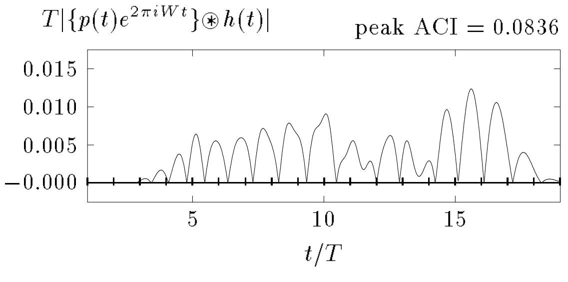
The use of the L_1 norm has greatly reduced the sidelobe level of $H(f)$ and significantly improved the ACI-energy rejection from 25.3 dB to 30.8 dB. Figure 2 shows how the magnitude response is partitioned between FIR and nonFIR portions of the receive filter.



The magnitude response of receive filter $H(f)$.



The ACI spectrum.



The complex magnitude of the ACI pulse.

$\boxed{2}$

As the peak amplitude at the lower end of the ACI spectrum is essentially fixed by the ISI constraints, major further reductions in ACI energy beyond the level reached in Example 2 would require suppression of that peak, which can only be done by reducing the sidelobe level of the spectrum of the transmitted pulse (as was done in [1]). The possibility of ameliorating the time-domain effects of this unavoidable ACI energy is the topic of the next section.

B. Minimizing Time-Domain ACI

The design examples below demonstrate two distinct approaches to minimization of ACI in the time domain. Both directly control the ACI pulse.

The Time-Domain ACI L_∞ Norm   The next example simply bounds an approximation to the peak amplitude of the ACI pulse:

Example $\boxed{3}$ This design uses the FF of Example 1, except for the following section:

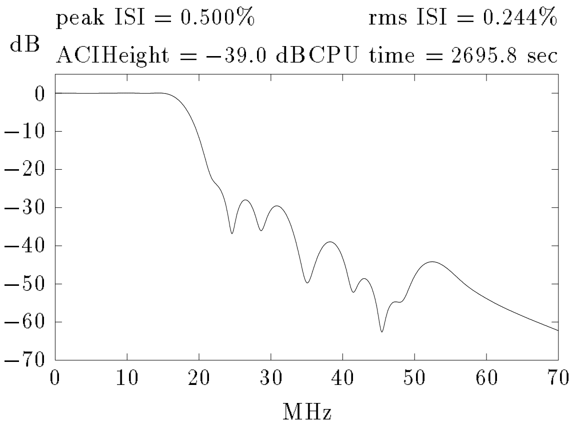
Adjacent-Channel Interference (ACI):

The ACI pulse $\kappa a(t)$ defined below approximates the inverse Fourier transform of $P(f - W)H(f)$.

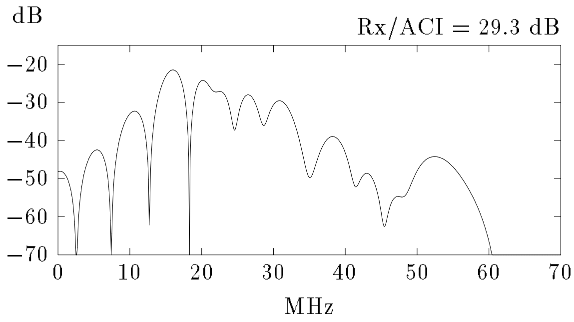
Let $W = 40$ MHz, let $df = \frac{1}{200T}$, and define $a(t) \stackrel{\Delta}{=} \frac{df}{\kappa} \times$

$$\sum_{k=\lfloor\frac{-W/2}{df}\rfloor}^{\lceil\frac{3W/2}{df}\rceil} P(k\,df - W)H(k\,df)e^{2\pi ik\,df\,t}.$$

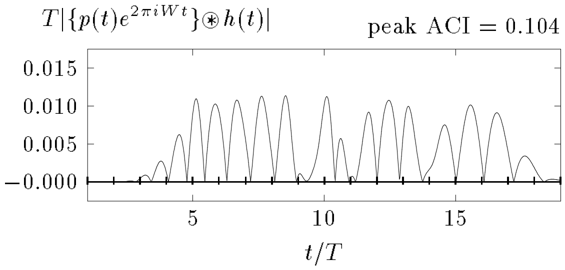
$\forall t \in [0, 20T]$ with $\Delta \leq 0.1T$,
require $\|a(t)\|_6 \leq \rho$.



The magnitude response of receive filter $H(f)$.



The ACI spectrum.



The complex magnitude of the ACI pulse.

$\boxed{3}$

By both measures, "Rx/ACI" energy and "peak ACI" (next paragraph), the Example 3 design is much better than that of Example 1, but not quite as good as that of Example 2. This motivates a more-sophisticated time-domain approach.

The Time-Domain Peak ACI If the constellation of possible complex symbols transmitted had a precisely circular perimeter of radius r_p, the largest possible ACI component at the receive-filter output, normalized by system gain κ, would be of magnitude $r_p \sum_n |a(\tau_a + nT)|$, for some worst-case τ_a, where τ_a represents the relative symbol timing of the unsynchronized transmit and receive systems. For noncircular constellations then, this

$$\text{"peak ACI"} \triangleq \max_{\tau_a \in (0,T]} \sum_n |a(\tau_a + nT)|$$

is still a convenient upper bound on worst-case ACI, because we can approximately express and optimize this sum in a linear program:

Example $\boxed{4}$ This design uses the FF of Example 3, except for the following sections:

Auxiliary optimization variables: The FF of Examples 1 and 3 is applicable here, except that the declaration of the ACI limit ρ is replaced with this:

Let σ be optimized
  and bounded below by 0.

Let $S = 10$, and
index ρ from 0 to $S-1$;

let $J = 9$,
for each $s = 0, \ldots, S-1$,
  index ρ_s from $-J$ to J, and
  let $\rho_{s,-J}, \ldots, \rho_{s,J}$ be optimized
    and bounded below by 0.

This makes ρ a two-dimensional array of optimization variables for use below.

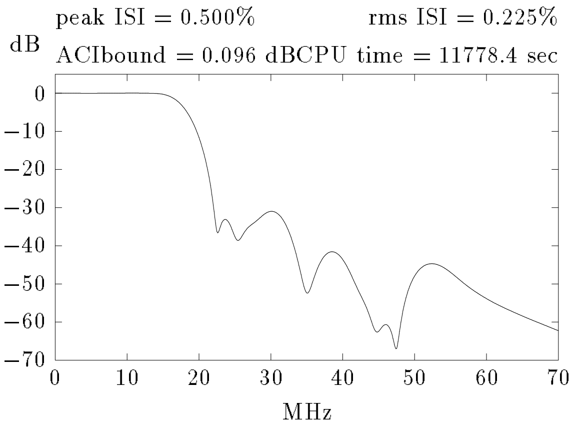
Adjacent-Channel Interference (ACI): The FF used here to control ACI is identical to that of Example 3 except that the last two lines ("$\forall t \in \ldots$" and "require...") of that code are replaced with the following:

Let $dt = T/S$;
for each $s = 0, \ldots, S-1$;
  for each $j = -J, \ldots, J$,
    require
    $\|a(\tau - s\,dt + jT)\|_6 \le \rho_{s,j}$; and
  require $\sum_{j=-J}^{J} \rho_{s,j} \le \sigma$.

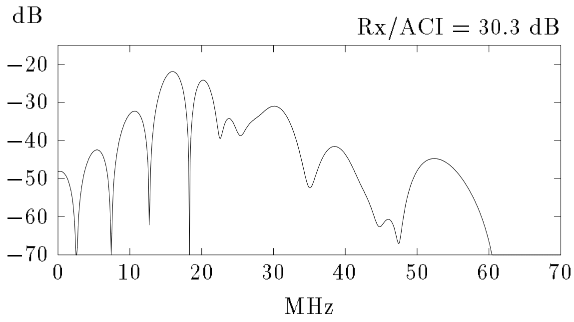
Optimization and Output: Minimize and output the bound σ on peak time-domain ACI:

Minimize σ, and
write σ to "ACIbound".
$\forall n = 0, \ldots, K-1$, write ψ_n to "Weights".

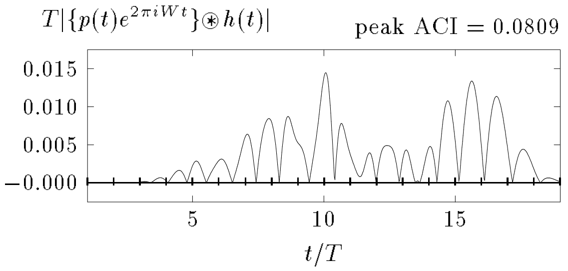
The peak ACI of 0.0809 only slightly bettered the best of the others, the 0.0836 of Example 2. Further, the Rx/ACI energy ratio here of 30.3 dB nearly matched the best such ratio, the 30.8 dB of Example 2.



The magnitude response of receive filter $H(f)$.



The ACI spectrum.



The complex magnitude of the ACI pulse.

4

III. SUMMARY AND CONCLUSIONS

With the proper software tools, linear programming becomes a powerful approach to the design of modem filters, making practical a wide variety of possible approaches to dealing with real-world design issues. Four linear-programming methods were presented here for the design of the FIR part of an analog/FIR hybrid receive filter with the goal of minimizing ACI: (1) minimizing peak ACI spectral height, (2) minimizing the L_1 norm of the ACI spectrum, (3) minimizing peak time-domain ACI-pulse height, and (4) minimizing worst-case time-domain ACI, approximated as the sum of the complex magnitudes of baud-spaced samples of the ACI pulse with the worst-case adjacent-channel timing epoch. Caution is advised, as these results are empirical, and the sample size is vanishingly small, but it appears that the second and fourth of these methods give excellent performance. It would be reasonable to expect that some back-and-forth iteration with the design of the transmit pulse [1] might significantly further improve ACI performance.

The `ffc` interpreter for the FF language is currently available by anonymous ftp from `ftp.ee.mtu.edu` in directory `/pub/jeffc`.

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